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# The First Run Preceded by a Quota*

A. J. Goldman and B. K. Bender

(March 20, 1962)

Let $a,b,c$ be positive integers with $b \leq \min(a, c-1)$. In an infinite sequence of independent trials with different success probabilities, define a "run" to consist of $a$ consecutive successes, and a "quota" to consist of $c$ or fewer consecutive trials containing $b$ or more successes. Recursion formulas are derived for the probabilities governing the first appearance of a run immediately preceded by a quota.

## 1. Introduction

This paper deals with the following problem, which arises in the analysis of certain target-detection systems. Consider an infinite sequence $\{T_n: n=1, 2, \ldots\}$ of independent probabilistic "trials." $T_n$ yields a "success" with probability $p_n$, and a "failure" with probability $q_n=1-p_n$. A *run* is defined to be a set of $a$ consecutive trials all yielding successes, where $a$ is a positive integer specified in advance. A *quota* is defined to be a set of $c$ consecutive trials, of which at least $b$ yield successes;[1] here $b$ and $c$ are positive integers specified in advance, with $b<c$. When does the first run which immediately follows a quota occur?

Of course the question can only be answered on a probabilistic basis, i.e., one seeks the probability, as a function of $n$, that $T_n$ is the last trial of the first such run. The analysis given below leads to a recursion equation for these probabilities (more precisely, for certain other closely related ones) under the additional hypothesis that $a \geq b$. The equation is somewhat unpleasant looking, but offers no particular difficulty as regards implementation on a digital computer.[2] Both the analysis and its result appear likely to be more complicated if $a<b$, so that it is fortunate that the hypothesis $a \geq b$ covers the applications we have in mind.

The special case $a=2, b=1, c=2$ has been solved by E. Brookner as part of a larger study;[3] the first author appreciates access to an advance copy of Brookner's paper. Since the methods used below are rather *ad hoc*, it should be noted that other work by Brookner[4] provides, in principle, a systematic approach to such problems; in particular, his methods can apparently be used to derive a solution in the case $a<b$.

The next section contains a precise statement of the problem, and a guide to the main results of the investigation.

## 2. Notation and Summary of Results

The probabilistic events of interest will be regarded as subsets of a *sample space*,[5] and so the notation of Boolean set-algebra will be employed. Thus the symbols $\cup$ and $\cap$ stand for set-theoretic *union* and *intersection*, respectively, while $A'$ will generically denote the complement of the event $A$. We depart from standard notation by using the symbols $\Sigma$ and "+" for the set-theoretic union of *disjoint* events (i.e., "events whose probabilities can simply be summed"). We set $p_n=0$ for $n<1$.

*Sponsored in part by the North American Air Defense Command (Directorate of Operations Analysis) and U.S. Army Air Defense Command.

[1] This definition must be slightly modified near the beginning of the sequence; see the definition of $Q_n$ in section 2.

[2] This assumes that a routine for computing the sequence $\{p_n\}$ is available.

[3] Eli Brookner, Cumulative probability of target detection for pulsed surveillance radars, Columbia University Electronics Research Laboratories Tech. Rept. T-10/122.

[4] Eli Brookner, Determination of the statistics of a recurrent event in a Markov chain, Columbia University Electronics Research Laboratory unpublished manuscript.

[5] For this approach to probabilistic events, see e.g., the standard reference by W. Feller, An introduction to probability theory and its applications (John Wiley & Sons, Inc., New York, N.Y.).

A set-theoretic union over an empty set of indices will be taken, by convention, to be the null event $\phi$; similarly a numerical sum over an empty set of indices will be taken as zero. A set-theoretic intersection over an empty set of indices will be taken as the "sure event" (i.e., the entire sample space), while a product over an empty set of indices will be assigned the value unity.

The following events will be involved in the analysis:

$S_n$: $T_n$ yielded a success ($S_n=\phi$ for $n<1$),
$S'_n$: $T_n$ yielded a failure,
$R_n=\bigcap_{0}^{a-1} S_{n-j}$: $T_n$ is the last trial of a *run* (of $a$ consecutive successes [6])

For the exact definition of "quota," it is convenient to define

$$m(n)=\max\ (n-c+1,1). \tag{1}$$

Then we have the definitions

$Q_n$: at least $b$ trials among $T_n$, $T_{n-1}$, . . ., $T_{m(n)}$ yield successes,
$E_n=R_n\cap Q_{n-a}$: the type of event whose first occurrence interests us.

Verbally, $E_n$ occurs if the $a$ trials numbered $n$, $n-1$, . . ., $n-a+1$ all yield successes, and in addition $b$ or more of the min $(c,n-a)$ trials numbered $n-a$, $n-a-1$, . . ., $m(n-a)$ yield successes. Still other events will be required and introduced later.

The generic notation $P(A)$ will be used for the probability of an event $A$. The probabilities of interest, those referring to the first trial which ends a run immediately preceded by a quota, are therefore designated

$$X_n=P\left(E_n\cap\bigcap_{i=1}^{n-1}E'_i\right) \tag{2}$$

It will be simpler to deal, instead, with the probability

$$Y_n=P\left(\bigcap_{i=1}^{n}E'_i\right) \tag{3}$$

that no run preceded by a quota occurs up to and including the trial $T_n$. The sequences $\{X_n\}$ and $\{Y_n\}$ are related by

$$Y_n=1-\sum_{i=1}^{n}X_i, \qquad X_n=Y_{n-1}-Y_n, \tag{4}$$

and the sequence $\{Y_n\}$ clearly obeys the "initial conditions"

$$Y_n=1 \qquad \text{if } n<a+b, \tag{5}$$

$$Y_{a+b}=1-\prod_{i=1}^{a+b}p_i. \tag{6}$$

We therefore assume $n>a+b$ in all that follows.

The reader is exhorted to aid himself, in following the derivations, with the diagram of an axis on which points corresponding to $T_n, T_{n-a+1}, T_{n-a}$, $T_{n-a-c+b+1}$, $T_{n-a-c+b}$ and $T_{n-a-c+1}$ are marked off.

The ultimate goal is a recursion equation for the sequence $\{Y_n\}$, while the method is the decomposition of $\bigcap_{i=1}^{n}E'_i$ into a union of disjoint events, each the intersection of certain independent events.[7] This analysis is carried out in section 3. The desired recursion equation,

[6] This run may be imbedded in a still longer sequence of consecutive successes.

[7] If the set of trials whose outcomes determine whether or not an event $A$ occurs is disjoint from the corresponding set of trials for event $B$, then $A$ and $B$ are clearly independent. This sufficient condition for independence is the one implicitly appealed to throughout the paper.

for $a+b<n\leq 2a+c$, is derived *without* the assumption $a\geq b$ and is given in eq (24). For $n>2a+c$, *under* the assumption $a\geq b$, it is given in eq (27). Both of these equations involve as coefficients certain auxiliary probabilities whose calculation is the subject of section 6. As for the case $n>2a+c$ and $a<b$, which is *not* solved in this paper, our best progress to date is reported in eq (31).

The assumption $a\geq b$ is made throughout sections 4, 5, and 7. In section 4 it is shown that the original recursion equation can be replaced by one which has higher order but fewer nonzero terms, and so may be more suitable for computation. The new equation is given by eq (34), which is valid for $n>2a+c$ and $b>1$, by eq (34a), which is valid for $a+b<n<2a+c$ and $b>1$, by eq (34b), which is valid for $n>2a+c$ and $b=0$, and by eq (34c), which is valid for $n>2a+c$ and $b=1$. The remaining cases ($n=2a+c$, or $n<2a+c$ and $b\leq 1$) are left to the reader.

Section 5 contains an examination of the special case in which $p_n$ is independent of $n$. The general recursion eq (24) becomes eqs (37) and (38), while the general recursion eq (27) becomes eq (39); for $b>1$, the general eq (34) yields eq (40). For this special case (i.e., $p_n=p$ and $b>1$) a method is given for calculating the mean occurrence time for the first run preceded by a quota; this is given compactly as eq (47). The extension to $b\leq 1$ offers no difficulty.

In section 7 we study the probability $Z_n$ that the first run preceded by a quota, if it ends at $T_n$, occurred as a sequence of $a+b$ consecutive successes. (Note that this is not the same as the probability that the first sequence of $a+b$ consecutive successes ends at $T_n$.) Equation (57), which is valid for $n>a+c+1$ and $b>0$, shows how the sequence $\{Z_n\}$ is determined by the sequences $\{X_n\}$ and $\{Y_n\}$. The illustrative case $b=1$, $c=2$ is investigated in some detail, and bounds on $Z_n$ are found in terms of $p_n$; see e.g., eq (61), valid for $n>3a+3$ and $a>1$, and eqs (62) and (63), valid for $n>6$ and $a=1$.

## 3. Analysis

In this section the hypothesis $a\geq b$ is not at first assumed. From $E_n=R_n\cap Q_{n-a}$, one has

$$E'_n=R'_n+(R_n\cap Q'_{n-a}),$$

where the convention for "disjoint union" has been used. Thus

$$\bigcap_{i=1}^{n} E'_i=E'_n\cap\bigcap_{i=1}^{n-1} E'_i=\Big(R'_n\cap\bigcap_{i=1}^{n-1} E'_i\Big)+\Big(R_n\cap Q'_{n-a}\cap\bigcap_{i=1}^{n-1} E'_i\Big). \tag{7}$$

The first disjunct on the right in eq (7) will now be decomposed further. Let $R'_{nk}$ be the event that the first failure among trials $T_n, T_{n-1}, \ldots,$ (in that order) occurs at $T_{n-k}$. Then $R'_n=\sum_{k=0}^{a-1} R'_{nk}$, which implies that [8]

$$R'_n\cap\bigcap_{i=1}^{n-1} E'_i=\sum_{k=0}^{a-1}\Big(R'_{nk}\cap\bigcap_{i=1}^{n-1} E'_i\Big). \tag{8}$$

However, it is easy to show the inclusion

$$R'_{nk}\subseteq E'_i \qquad \text{for } n-k\leq i\leq n-1,\ 1\leq k\leq a-1; \tag{9}$$

for if $R'_{nk}$ occurs then so does $S'_{n-k}$, and obviously this precludes the occurrence of $R_i$ (and thus of $E_i$) for $n-k\leq i$ and $k\leq a-1$. By (8) and (9), we have the desired decomposition [8a]

---

[8] The distributive law of Boolean set-algebra is to be used. It will be employed at other points without explicit mention.

[8a] Note the use, for $k=0$, of our convention about intersections over an empty set of indices.

$$R'_n \cap \bigcap_{i=1}^{n-1} E'_i = \sum_{k=0}^{a-1} \left( R'_{nk} \cap \bigcap_{i=n-k}^{n-1} E'_i \cap \bigcap_{i=1}^{n-k-1} E'_i \right)$$
$$= \sum_{k=0}^{a-} \left( R'_{nk} \cap \bigcap_{i=1}^{n-k-1} E'_i \right). \tag{10}$$

Now $R'_{nk}$ and $\bigcap_{i=1}^{n-k-1} E'_i$ are independent, and furthermore

$$P(R'_{nk}) = q_{n-k} \prod_{j=0}^{k-1} p_{n-j}, P\left( \bigcap_{i=1}^{n-k-1} E'_i \right) = Y_{n-k-1}. \tag{11}$$

Equations (7), (10), and (11) therefore yield

$$Y_n = \sum_{k=0}^{a-1} q_{n-k} \left( \prod_{j=0}^{k-1} p_{n-j} \right) Y_{n-k-1} + P\left( R_n \cap Q'_{n-a} \cap \bigcap_{i=1}^{n-1} E'_i \right). \tag{12}$$

Equation (12) indicates that further progress requires a decomposition of the event $Q'_{n-a} \cap \bigcap_{i=1}^{n-1} E'_i$. As a first step in this direction, it will be shown that

$$Q'_{n-a} \subseteq E'_i \qquad \text{for } n-a \leq i \leq n-1, \tag{13}$$

which implies that

$$Q'_{n-a} \cap \bigcap_{i=1}^{n-1} E'_i = Q'_{n-a} \cap \bigcap_{i=n-a}^{n-1} E'_i \cap \bigcap_{i=1}^{n-a-1} E'_i = Q'_{n-a} \cap \bigcap_{i=1}^{n-a-1} E'_i$$

and thus that

$$R_n \cap Q'_{n-a} \cap \bigcap_{i=1}^{n-1} E'_i = R_n \cap Q'_{n-a} \cap \bigcap_{i=1}^{n-a-1} E'_i. \tag{14}$$

Since (a) $R_n$ is independent of $Q'_{n-a}$ and also of $E'_i$ for $i \leq n-a$, and (b) $P(R_n) = \prod_{j=0}^{a-1} p_{n-j}$, application of eq (14) to eq (12) yields

$$Y_n = \sum_{k=0}^{a-1} q_{n-k} \left( \prod_{j=0}^{k-1} p_{n-j} \right) Y_{n-k-1} + \left( \prod_{j=0}^{a-1} p_{n-j} \right) P\left( Q'_{n-a} \cap \bigcap_{i=1}^{n-a-1} E'_i \right). \tag{15}$$

To prove (13), assume that $Q'_{n-a}$ and $E_i$ both hold with $n-a \leq i \leq n-1$. Then $R_i$ holds, so that $i \geq a+b$ and the trials $T_i, T_{i-1}, \ldots, T_{i-a+1}$ all yield successes; in particular, this holds for the trials

$$T_{n-a}, T_{n-1-a}, \ldots, T_{i+1-a}. \tag{16}$$

If $i \leq n-b$, then $T_{n-a}, T_{n-a-1}, T_{n-a-2}, \ldots, T_{n-a-b+1}$ would be in the sequence (16) and therefore would all yield successes, contradicting the occurrence of $Q'_{n-a}$. So $i > n-b$, i.e., $i-a > n-a-b > n-a-c$, a fact to be used below in (17), in the form $i-a \geq m(n-a)$.

Since $E_i$ occurs, $Q_{i-a}$ must occur, so that there are at least $b$ successes in the trials $T_{i-a}, T_{i-a-1}, \ldots, T_{m(i-a)}$. This last-mentioned sequence of trials can be split into two subsequences,

$$T_{i-a}, T_{i-a-1}, \ldots, T_{m(n-a)}, \tag{17}$$

and

$$T_{n-a-c}, T_{n-a-c-1}, \ldots, T_{m(i-a)} \tag{18}$$

where the second subsequence is absent if $n \leq a+c$. Of the "at least $b$ successes" just mentioned, at most $n-i$ can occur in the subsequence (18), since the number of trials in this subsequence is at most

$$n-a-c-(i-a-c+1)+1 = n-i.$$

Thus there are *at least* $b-(n-i)$ successes in the subsequence (17). Combining these with the $n-i$ successes in the sequence (16) (which is disjoint from (17)) yields at least $b$ successes in the sequence $T_{n-a}, T_{n-a-1}, \ldots, T_{m(n-a)}$, but this contradicts the occurrence of $Q'_{n-a}$. Thus the assumption that $Q'_{n-a}$ and $E_i$ both hold is untenable, and the inclusion (13) is proved.

Equation (15) shows that further progress requires a decomposition of the event $Q'_{n-a} \cap \bigcap_{i=1}^{n-a-1} E'_i$. This will first be done for $n \le 2a+c$, by showing that

$$Q'_{n-a} \subseteq E'_i \qquad \text{for } 1 \le i \le n-a-1, \qquad \text{if } a+b < n \le a+c, \tag{19}$$

$$Q'_{n-a} \subseteq E'_i \qquad \text{for } n-a-c+1 \le i \le n-a-1, \qquad \text{if } a+c < n \le 2a+c. \tag{20}$$

These results imply the decompositions

$$Q'_{n-a} \cap \bigcap_{i=1}^{n-a-1} E'_i = Q'_{n-a} \qquad \text{if } a+b < n \le a+c, \tag{21}$$

$$Q'_{n-a} \cap \bigcap_{i=1}^{n-a-1} E'_i = Q'_{n-a} \cap \bigcap_{i=1}^{n-a-c} E'_i \qquad \text{if } a+c < n \le 2a+c. \tag{22}$$

Since $E_i$ cannot occur for $i<a$, and can occur for $i=a$ only when $b=0$, in which case we adopt the convention $c=0$, the last two displays can be rewritten as

$$Q'_{n-a} \cap \bigcap_{i=1}^{n-a-1} E'_i = Q'_{n-a} \qquad \text{if } a+b < n \le 2a+c.$$

Substitution of these results into eq (15) yields

$$Y_n = \sum_{k=0}^{a-1} q_{n-k} \left( \prod_{j=0}^{k-1} p_{n-j} \right) Y_{n-k-1} + \left( \prod_{j=0}^{a-1} p_{n-j} \right) P(Q'_{n-a}) \qquad \text{for } a+b < n \le 2a+c. \tag{24}$$

The evaluation of $P(Q'_{n-a})$, which appears in eq (24), will be discussed later.

To prove the inclusions (19) and (20), first note that both can be simultaneously written as

$$Q'_{n-a} \subseteq E'_i \qquad \text{for } m(n-a) \le i \le n-a-1, \qquad \text{if } a+b < n \le 2a+c.$$

Suppose, on the contrary, that both $Q'_{n-a}$ and $E_i$ occur. Since $E_i$ occurs, there must be at least $a+b$ successes in the sequence of trials $T_i, T_{i-1}, \ldots, T_{m(i-a)}$. Since

$$i \le n-a-1 \le (2a+c)-a-1 = a+c-1$$

we have $m(i-a)=1$, and so there are at least $a+b$ successes among the trials $T_i, T_{i-1}, \ldots, T_1$. Since $Q'_{n-a}$ holds, at most $b-1$ of these successes occur among trials $T_i, T_{i-1}, \ldots, T_{m(n-a)}$, and so there are at least $a+b-(b-1)=a+1$ successes in trials $T_{m(n-a)-1}, T_{m(n-a)-2}, \ldots, T_1$. Thus

$$a+1 \le m(n-a)-1 = \max\,(n-a-c, 0),$$

implying $a+1 \le n-a-c$ and thus $n > 2a+c$, a contradiction. So (19) and (20) are proved. Thus the desired recursion equation for the sequence $\{Y_n\}$ has been derived for $n \le 2a+c$, and is given by eqs (5), (6), and (24). From now on, $n > 2a+c$ is assumed.

To decompose $Q'_{n-a} \cap \bigcap_{i=1}^{n-a-1} E'_i$ when $n > 2a+c$, it is helpful to observe that $Q'_{n-a}$ can be defined as the occurrence of at least $c-(b-1)=(c-b)+1$ failures in the $c$ trials between $T_{n-a}$ and $T_{n-a-c+1}$ inclusive. Not all of these failures can occur in the $c-b$ trials between $T_{n-a}$ and $T_{n-a-(c-b)+1}$ inclusive. We therefore introduce the new events defined for $0 \le t \le b-1$ by

$F_{nt}$: the $(c-b)+1$st failure among $T_{n-a}, T_{n-a-1}, \ldots,$

$T_{n-a-c+1}$ (in that order) occurs at $T_{n-a-(c-b)-t}$.

Then $Q'_{n-a} = \sum_{t=0}^{b-1} F_{nt}$, which implies the decomposition

$$Q'_{n-a}\cap\bigcap_{i=1}^{n-a-1}E'_i=\sum_{t=0}^{b-1}\left(F_{nt}\cap\bigcap_{i=1}^{n-a-1}E'_i\right),$$

so that eq (15) becomes

$$Y_n=\sum_{k=0}^{a-1}q_{n-k}\left(\prod_{j=0}^{k-1}p_{n-j}\right)Y_{n-k-1}+\left(\prod_{j=0}^{a-1}p_{n-j}\right)\sum_{t=0}^{b-1}P\left(F_{nt}\cap\bigcap_{i=1}^{n-a-1}E'_i\right). \tag{25}$$

The solution under the *extra* hypothesis $a\geq b$ will now be *completed* by showing that

$$F_{nt}\subseteq E'_i \qquad \text{for } n-a-(c-b)-t\leq i\leq n-a-1, \qquad \text{if } a\geq b. \tag{26}$$

This will imply

$$F_{nt}\cap\bigcap_{i=1}^{n-a-1}E'_i=F_{nt}\cap\bigcap_{i=1}^{n-a-(c-b)-t-1}E'_i$$

which, since $F_{nt}$ and $E'_i$ are independent for $1\leq i\leq n-a-(c-b)-t-1$, will turn eq (25) into

$$Y_n=\sum_{k=0}^{a-1}q_{n-k}\left(\prod_{j=0}^{k-1}p_{n-j}\right)Y_{n-k-1}+\left(\prod_{j=0}^{a-1}p_{n-j}\right)\sum_{t=0}^{b-1}P(F_{nt})Y_{n-a-(c-b)-t-1} \qquad \text{if } a\geq b. \tag{27}$$

Equation (27) is the desired recursion relation for $n>2a+c$; together with eq (24) and the initial conditions of eqs (5) and (6), it entirely specifies the sequence $\{Y_n\}$ when $a\geq b$. The calculation of the coefficients $P(F_{nt})$ will be discussed later.

To prove the inclusion (26), suppose that $F_{nt}$ and $E_i$ both hold, with $a\geq b$ and $n-a-(c-b)-t\leq i\leq n-a-1$. Since $i\geq n-a-(c-b)-t$, the (success-yielding) trials between $T_i$ and $T_{i-a+1}$ inclusive must have greater subscripts than the (failure-yielding) $T_{n-a-(c-b)-t}$ so that

$$n-a-c+1\leq n-a-(c-b)-t<i-a+1\leq i\leq n-a-1$$

so that the $a$ successes just mentioned fall between $T_{n-a}$ and $T_{n-a-c+1}$ inclusive. Because $a\geq b$, this implies that $Q_{n-a}$ occurs, contradicting the assumption that $F_{nt}$ occurs (recall that $F_{nt}\subseteq Q'_{n-a}$).

The reasoning just employed carries over in part to the case $a\leq b$. Specifically, it will be shown that

$$F_{nt}\supseteq E'_i \qquad \text{for } n-a-(c-b)-t\leq i\leq n-a-1 \text{ and } t\leq a-1 \tag{28}$$

so that eq (25) becomes

$$Y_n=\sum_{k=0}^{a-1}q_{n-k}\left(\prod_{j=0}^{k-1}p_{n-j}\right)Y_{n-k-1}+\left(\prod_{j=0}^{a-1}p_{n-j}\right)\sum_{t=0}^{a-1}P(F_{nt})Y_{n-a-(c-b)-t-1}$$
$$+\left(\prod_{j=0}^{a-1}p_{n-j}\right)\sum_{t=a}^{b-1}P\left(F_{nt}\cap\bigcap_{i=1}^{n-a-1}E'_i\right). \tag{29}$$

To prove the inclusion (28), suppose that $F_{nt}$ and $E_i$ both hold. As in the last paragraph, $E_i$ requires at least $a$ successes between trials $T_{n-a}$ and $T_{n-a-(c-b)-t}$ inclusive, whereas $F_{nt}$ requires exactly $(c-b)+1$ failures (and thus exactly t successes) in these $(c-b)+t+1$ trials. Therefore, $a\leq t$, contrary to hypothesis in (28).

It is also easy to show that, for $a\leq t\leq b-1$,

$$F_{nt}\subseteq E'_i \qquad \text{for } n-a-(c-b)-t\leq i\leq n-(c-b)-t-1, \tag{30}$$

so that eq (29) becomes

$$Y_n=\sum_{k=0}^{a-1}q_{n-k}\left(\prod_{j=0}^{k-1}p_{n-j}\right)Y_{n-k-1}+\left(\prod_{j=0}^{a-1}p_{n-j}\right)\sum_{t=0}^{a-1}P(F_{nt})Y_{n-a-(c-b)-t-1}$$
$$+\left(\prod_{j=0}^{a-1}p_{n-j}\right)\sum_{t=a}^{b-1}P\left(F_{nt}\cap\bigcap_{i=n-(c-b)-t}^{n-a-1}E'_i\cap\bigcap_{i=1}^{n-a-(c-b)-t-1}E'_i\right). \tag{31}$$

for if $F_{nt}$ and $E_i$ both occur, with $i \geq n-a-(c-b)-t$, then the fact that $T_i$, $T_{i-1}$, . . ., $T_{i-a+1}$ all yield successes whereas $T_{n-a-(c-b)-t}$ yields a failure, implies that

$$i-a+1 > n-a-(c-b)-t,$$

i.e., $i \geq n-(c-b)-t$. The authors have not succeeded in carrying this analysis further in the case $a<b$.

## 4. A Possible Simplification

The recursion eq (27) is of order $a+c$, i.e., each member of the sequence $\{Y_n\}$ is expressed in terms of the $a+c$ members immediately preceding it. Actually, only $a+b$ of these $a+c$ preceding members are employed,[9] the others appearing with zero coefficients in eq (27). In this section eq (27) is used to derive a recursion equation of the higher degree $a+c+1$, which however has the advantage that calculation of a particular member of $\{Y_n\}$ requires only $b+3$ of the preceding $a+c+1$ members (and only 2 if $b=0$). Of course a detailed evaluation of this "advantage" would involve a comparison of the computational labor for the coefficients of eq (27) versus that for the coefficients of the new equation.

For the derivation, first change $n$ to $n+1$ in eq (27) and shift "dummy indices" to write the result, for $b>1$, as

$$Y_{n+1}=q_{n+1}Y_n+\sum_{k=0}^{a-2} q_{n-k}\left(\prod_{j=0}^{k} p_{n+1-j}\right)Y_{n-k-1}+\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)\left(\prod_{i=0}^{c-b} q_{n+1-a-i}\right)Y_{n-a-(c-b)}$$
$$+\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)P(F_{n+1,1})Y_{n-a-(c-b)-1}+\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)\sum_{t=1}^{b-2} P(F_{n+1,t+1})Y_{n-a-(c-b)-t-1}. \qquad (32)$$

Next multiply eq (27) through by $p_{n+1}$, and write the result as

$$0=p_{n+1}Y_n-\sum_{k=0}^{a-2} q_{n-k}\left(\prod_{j=0}^{k} p_{n+1-j}\right)Y_{n-k-1}-q_{n-a+1}\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)Y_{n-a}$$
$$-\left(\prod_{j=0}^{a} p_{n+1-j}\right)\left(\prod_{i=0}^{c-b} q_{n-a-i}\right)Y_{n-a-(c-b)-1}-\left(\prod_{j=0}^{a} p_{n+1-j}\right)\sum_{t=1}^{b-1} P(F_{nt})Y_{n-a-(c-b)-t-1}. \qquad (33)$$

Now add eqs (32) and (33), noting that $p_{n+1}+q_{n+1}=1$:

$$Y_{n+1}=Y_n-q_{n-a+1}\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)Y_{n-a}+\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)\left(\prod_{i=0}^{c-b} q_{n+1-a-i}\right)Y_{n-a-(c-b)}$$
$$+\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)\left[P(F_{n+1,1})-p_{n+1-a}\left(\prod_{i=0}^{c-b} q_{n-a-i}\right)\right]Y_{n-a-(c-b)-1},$$
$$+\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)\sum_{t=1}^{b-2}\left[P(F_{n+1,t+1})-p_{n+1-a}P(F_{nt})\right]Y_{n-a-(c-b)-t-1}$$
$$-\left(\prod_{j=0}^{a} p_{n+1-j}\right)P(F_{n,b-1})Y_{n-a-c}. \qquad (34)$$

This is the desired equation.

A similar equation can be derived to replace eq (24). The procedure is as above, and the result is:

$$Y_{n+1}=Y_n-q_{n-a+1}\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)Y_{n-a}+\left(\prod_{j=0}^{a-1} p_{n+1-j}\right)[P(Q'_{n+1-a})-p_{n+1-a}P(Q'_{n-a})] \qquad (34a)$$

for $a+b<n<2a+c$. The analogous equation for the "transition case" $n=2a+c$ will be omitted.

[9] The order of the recursion is related to the "memory" requirements of a possible computer program, while the number of nonzero coefficients is relevant to the amount of computation required for each $Y_n$.

If $b=0$ or $b=1$, the preceding equations must be modified. For $b=0$, eq (34) becomes

$$Y_{n+1}=Y_n-q_{n-a+1}\left(\prod_{j=0}^{a-1}p_{n+1-j}\right)Y_{n-a}, \tag{34b}$$

while for $b=1$ it becomes

$$Y_{n+1}=Y_n-q_{n-a+1}\left(\prod_{j=0}^{a-1}p_{n+1-j}\right)Y_{n-a}+\left(\prod_{j=0}^{a-1}p_{n+1-j}\right)\left(\prod_{i=0}^{c-b}q_{n+1-a-i}\right)Y_{n-a-c+1}$$
$$-\left(\prod_{j=0}^{a}p_{n+1-j}\right)\left(\prod_{i=0}^{c-b}q_{n-a-i}\right)Y_{n-a-c}. \tag{34c}$$

The analogous results for eq (34a) are omitted.

## 5. A Special Case

It is natural to inquire what form the recursion equations take in the relatively simple case when all the trials are governed by the same probability distribution, i.e.,

$$p_n=p,\ q_n=q=1-p.$$

The initial conditions are

$$Y_n=1 \qquad \text{for } n<a+b, \tag{35}$$

$$Y_{a+b}=1-p^{a+b}. \tag{36}$$

From the definition of $Q'_{n-a}$ it follows that in eq (24)

$$P(Q'_{n-a})=\sum_{t=0}^{b-1}\binom{n-a}{t}p^tq^{n-a-t} \qquad \text{for } a+b<n\leq a+c,$$

$$P(Q'_{n-a})=\sum_{t=0}^{b-1}\binom{c}{t}p^tq^{c-t} \qquad \text{for } a+c<n\leq 2a+c.$$

Thus the early recursion equations read (for $q>0$)

$$Y_n=q\sum_{k=0}^{a-1}p^kY_{n-k-1}+p^aq^{n-a}\sum_{t=0}^{b-1}\binom{n-a}{t}p^tq^{-t} \qquad \text{if } a+b<n\leq a+c, \tag{37}$$

$$Y_n=q\sum_{k=0}^{a-1}p^kY_{n-k-1}+p^aq^c\left(\sum_{t=0}^{b-1}\binom{c}{t}p^tq^{-t}\right)Y_{n-a-c} \qquad \text{if } a+c<n\leq 2a+c. \tag{38}$$

From the definition of $F_{nt}$, it follows that

$$P(F_{nt})=\binom{c-b+t}{t}p^tq^{c-b+1},$$

so that eq (27), which is valid if $a\geq b$, becomes

$$Y_n=q\sum_{k=0}^{a-1}p^kY_{n-k-1}+p^aq^{c-b+1}\sum_{t=0}^{b-1}\binom{c-b+t}{t}p^tY_{n-a-(c-b)-t-1} \qquad \text{if } n>2a+c, \tag{39}$$

while eq (34) becomes (for $b>1$)

$$Y_{n+1}=Y_n-qp^aY_{n-a}+p^aq^{c-b+1}Y_{n-a-(c-b)}+(c-b)p^{a+1}q^{c-b+1}Y_{n-a-(c-b)-1}$$
$$+p^{a+1}q^{c-b+1}\sum_{t=1}^{b-2}\binom{c-b+t}{t+1}p^tY_{n-a-(c-b)-t-1}-p^{a+b}q^{c-b+1}\binom{c-1}{b-1}Y_{n-a-c}. \tag{40}$$

Let the random variable $N$ be defined by the condition that $T_N$ is the last trial of the first run immediately preceded by a quota. The sequence $\{X_n\}$ defined in eq (2) is thus

precisely the frequency distribution function of $N$. Although the equations above are not spectacularly simple despite the simple choice of the sequence $\{p_n\}$, they at least permit description of a finite procedure for calculating the expected value $E(N)$ of $N$ in the special case under consideration (with $a \geq b$).

To this end, first use eq (4) to write

$$\begin{aligned} E(N) &= \sum_1 n X_n = X_1 + \sum_2 n(Y_{n-1} - Y_n) \\ &= (1 - Y_1) + \lim_{m \to \infty} \left( \sum_2^m n Y_{n-1} - \sum_2^m n Y_n \right) \\ &= 1 + \lim_{m \to \infty} \left( \sum_1^{m-1} Y_n - m Y_m \right). \end{aligned}$$

Since the definition (3) of the sequence $\{Y_n\}$ ensures that this is a monotone nonascending sequence of nonnegative terms, a classical theorem on infinite series [10] can be invoked to assert that $mY_m \to 0$ if $\sum_1 Y_n < \infty$. This yields

$$E(N) = 1 + \sum_1 Y_n \qquad \text{if } \sum_1 Y_n < \infty, \tag{41}$$

so that the evaluation of $E(N)$ reduces to that of $\sum_1 Y_n$. This of course holds even in the general case.

We now employ the standard formal procedure for summing an infinite series whose terms obey a linear difference equation with constant coefficients. Write eq (39) briefly as

$$Y_n = \sum_{i=1}^{a+c} a_i Y_{n-i} \qquad \text{for } n > 2a + c, \tag{42}$$

and introduce the generating function

$$g(z) = \sum_1 Y_n z^n \tag{43}$$

as well as the polynomial

$$f(z) = 1 - \sum_{i=1}^{a+c} a_i z^i. \tag{44}$$

An elementary calculation using eq (42) yields

$$\begin{aligned} f(z)g(z) &= \sum_1 Y_n z^n - \sum_{i=1}^{a+c} a_i \sum_1 Y_n z^{n+i} \\ &= \sum_1 Y_n z^n - \sum_{i=1}^{a+c} a_i \sum_{n=i+1} Y_{n-i} z^n \\ &= \sum_1 Y_n z^n - \sum_{n=2} \left( z^n \sum \{ a_i Y_{n-i} : 1 \leq i \leq \min(a+c, n-1) \} \right) \\ &= 1 + \sum_2^{a+c} \left( Y_n - \sum_{i=1}^{n-1} a_i Y_{n-i} \right) z^n + \sum_{a+c+1} \left( Y_n - \sum_{i=1}^{a+c} a_i Y_{n-i} \right) z^n \\ &= 1 + \sum_2^{a+c} \left( Y_n - \sum_{i=1}^{n-1} a_i Y_{n-i} \right) z^n + \sum_{a+c+1}^{2a+c} \left( Y_n - \sum_{i=1}^{a+c} a_i Y_{n-i} \right) z^n = h(z), \end{aligned} \tag{45}$$

where the coefficients of $h(z)$ can be found from eq (35) to (38).

It will be shown that for $0 < p < 1$

$$f(z) \neq 0 \qquad \text{for } |z| \leq 1. \tag{46}$$

[10] See p. 124 of Knopp, Theory and Application of Infinite Series, Blackie (1951 edition).

This justifies the preceding formal manipulations for $|z| \leq 1$, permits solution of eq (45) as $g(z)=h(z)/f(z)$, and permits use of eq (43) to write $\sum_i Y_n=g(1)$, so that eq (41) becomes

$$E(N)=1+h(1)/f(1), \tag{47}$$

the promised formula for $E(N)$. In connection with eq (47), note that by eqs (39) and (44)

$$f(1)=p^a\left(1-q^{c-b+1}\sum_{t=0}^{b-1}\binom{c-b+t}{t}p^t\right). \tag{48}$$

To prove the nonequality (46), note that all $a_i$ in eqs (42) and (39) are nonnegative, so that for $|z| \leq 1$,

$$\left|\sum_{i=1}^{a+c} a_i z^i\right| \leq \sum_{i=1}^{a+c} a_i |z|^i \leq \sum_{i=1}^{a+c} a_i;$$

thus (46) is true if $\sum_{i=1}^{a+c} a_i < 1$, i.e., if $f(1)>0$. By eq (48) this is equivalent to

$$\sum_{t=0}^{b-1}\binom{c-b+t}{t}p^t q^{c-b+1} < 1,$$

which is true since the left-hand sum is the value at $b-1$ of the cumulative negative binomial probability distribution with parameters $c-b+1$ and $q$. (See Feller, *op. cit.* in footnote 5.)

As in section 4, the cases $b=0$ and $b=1$ require special treatment, which is left to the reader.

## 6. Auxiliary Probabilities

This section deals with the probabilities $P(Q'_{n-a})$, which appear in eq (24), and $P(F_{nt})$, which appear in eq (27). The "best" way of calculating these quantities (or more generally, of arranging the whole recursive computation) is likely to depend strongly on the values of $a,b,c$, on the peculiarities of the specific sequence $\{p_n\}$ of success probabilities under consideration, and perhaps on special characteristics of the calculating equipment used. Thus the procedure sketched below is illustrative, rather than "recommended."

For $a+b<n \leq a+c$, $Q'_{n-a}$ is the event that the sequence of $n-a$ trials $T_{n-a}, T_{n-a-1}, \ldots, T_1$ yield at least $n-a-(b-1)$ failures. Not all of these failures can occur in the $n-a-b$ trials between $T_{n-a}$ and $T_{b+1}$, respectively. We therefore introduce new events defined for $0 \leq t \leq b-1$ by

$G_{nt}$: the $n-a-(b-1)^{st}$ failure among $T_{n-a}, T_{n-a-1}, \ldots, T_1$ (in that order) occurs at $T_{b-t}$.

Then

$$Q'_{n-a}=\sum_{t=0}^{b-1} G_{nt}, \text{ so that}$$

$$P(Q'_{n-a})=\sum_{t=0}^{b-1} P(G_{nt}) \qquad \text{for } a+b<n \leq a+c. \tag{49}$$

As in the analysis before eq (25),

$$P(Q'_{n-a})=\sum_{t=0}^{b-1} P(F_{nt}) \qquad \text{for } a+c<n \leq 2a+c. \tag{50}$$

The events $F_{nt}$ and $G_{nt}$ are special cases of the event

$H_{nkt}$: the $k$th failure among $T_n, T_{n-1}, \ldots, T_1$ (in that order) occurs at $T_{n-k+1-t}$,

defined for $0 \leq t \leq b-1$ and $1 \leq k \leq n-b+2$. Specifically,

$$G_{nt}=H_{n-a,n-a-b+1,b},\ F_{nt}=H_{n-a,c-b+1,b}.$$

Thus it suffices to consider the probabilities $P(H_{nkt})$. Now introduce the event

$$J_{nkt}\text{: precisely } k-1 \text{ failures between } T_n \text{ and } T_{n-k+2-t} \text{ inclusive.}$$

Then clearly $P(H_{n10})=q_n$ and $P(H_{nkt})=q_{n-k+1-t}P(J_{nkt})$ otherwise, so it suffices to consider the probabilities $P(J_{nkt})$ for $(k,t)\neq(1,0)$.

These probabilities can be computed by recursion on $k$ and $t$ after disposing of the case $k=1$ by

$$P(J_{n1t})=\prod_{i=0}^{t-1} p_{n-i} \qquad \text{for } t>0.$$

The initiation of the recursion, for fixed $k>1$, is

$$P(J_{nk0})=\prod_{i=0}^{k-2} q_{n-i} \qquad \text{for } k>1,$$

and the "recursive step" is

$$P(J_{nkt})=p_{n-k+2-t}P(J_{n,k,t-1})+q_{n-k+2-t}P(J_{n,k-1,t-1}) \qquad \text{for } k>1,\ t>1.$$

## 7. A Related Problem

Let $K_n$ be the event that $T_n$ is the last of a sequence of $a+b$ consecutive trials all yielding successes. That is,

$$K_n=\bigcap_{i=0}^{a+b-1} S_{n-i}. \tag{51}$$

For the application which motivated this paper, it is of interest to consider the conditional probabilities

$$Z_n=P\left(K_n/E_n\cap\bigcap_{i=1}^{n-1}E_i'\right) \tag{52}$$

defined for $n\geq a+b$. From the definition of conditional probability, we have [11]

$$X_nZ_n=P\left(K_n\cap E_n\cap\bigcap_{i=1}^{n-1}E_i'\right),$$

which because of the inclusion $K_n\subseteq E_n$ can be rewritten

$$X_nZ_n=P\left(K_n\cap\bigcap_{i=1}^{n-1}E_i'\right). \tag{53}$$

Since the preceding material in principle permits the evaluation of $X_n$ when $a\geq b$, we need only investigate the (unconditional) probability on the right-hand side of eq (53). It will be assumed that $n>a+c+1$ and $b>0$.

It will be shown below that

$$K_n\cap E_{n-a}'=K_n\cap\bigcap_{i=n-a-c}^{n-a-b}S_i', \tag{54}$$

which implies that

$$K_n\cap\bigcap_{i=1}^{n-1}E_i'=K_n\cap\bigcap_{i=n-a-c}^{n-a-b}S_i'\cap\bigcap_{i=1}^{n-2}E_i'.$$

[11] Recall the definition of $X_n$ in eq (2).

Since $S_i' \subseteq E_i'$, the last equation can be rewritten

$$K_n \cap \bigcap_{i=1}^{n-1} E_i' = K_n \cap \bigcap_{i=n-a-c}^{n-a-b} S_i' \cap \bigcap_{i=n-a-b+1}^{n-2} E_i' \cap \bigcap_{i=1}^{n-a-c-1} E_i'. \tag{55}$$

To prove eq (54), first assume its left-hand event occurs. Tentatively suppose $S_i$ occurs, with $n-a-c \le i \le n-a-b$. Since $K_n$ occurs, trials $T_{n-1}$ through $T_{n-a}$ all yield successes (i.e., $R_{n-1}$ holds), and in addition the $b-1$ trials $T_{n-a-1}$ through $T_{n-a-b+1}$ yield $b-1$ successes. These $(b-1)$ successes, together with $S_i$, occur between $T_{n-a-1}$ and $T_{n-a-c}$ inclusive, and so $Q_{n-1-a}$ holds. Thus $E_{n-1} = R_{n-1} \cap Q_{n-1-a}$ holds, contradicting the assumption that the left-hand event in eq (54) holds. So the tentative supposition is untenable, and the left-hand event is contained in (i.e., implies) the right-hand one. Conversely, if the right-hand event holds then trials $T_{n-a-b}$ through $T_{n-a-c}$ inclusive constitute $(c-b)+1$ failures between $T_{n-a-1}$ and $T_{n-a-c}$ inclusive, so that $Q_{n-1-a}'$, and thus $E_{n-1}'$, must hold. This shows that the right-hand event contains the left-hand one, and so eq (54) is proved.

Next it will be shown that

$$\bigcap_{i=n-a-c}^{n-a-b} S_i' \subseteq E_i' \qquad \text{for } n-a-b+1 \le i \le n-2. \tag{56}$$

This implies that eq (55) can be rewritten

$$K_n \cap \bigcap_{i=1}^{n-1} E_i' = K_n \cap \bigcap_{i=n-a-c}^{n-a-b} S_i' \cap \bigcap_{i=1}^{n-a-c-1} E_i',$$

and since the three events on the right-hand side are independent, eq (53) becomes

$$\begin{aligned} X_n Z_n &= P(K_n) P\left(\bigcap_{i=n-a-c}^{n-a-b} S_i'\right) P\left(\bigcap_{i=1}^{n-a-c-1} E_i'\right) \\ &= \left(\prod_{i=0}^{a+b-1} p_{n-i}\right)\left(\prod_{i=a+b}^{a+c} q_{n-i}\right) Y_{n-a-c-1}, \end{aligned} \tag{57}$$

thus completing the solution.

To prove the inclusion (56), suppose its left-hand event occurs, and assume $n-a-b+1 \le i \le n-2$. Tentatively assume $E_i$ occurs. Then $T_i$ through $T_{i-a+1}$ inclusive must yield successes, and so (since $i > n-a-b$) must have subscripts greater than the failure-yielding $T_{n-a-b}$. Thus $i-a+1 > n-a-b$, and so $n-b \le i \le n-2$. This implies that

$$i-a-c+1 < n-a-c < n-a-b \le i-a,$$

so that the $(c-b)+1$ failure-yielding trials $T_{n-a-b}$ through $T_{n-a-c}$ inclusive lie between $T_{i-a}$ and $T_{i-a-c+1}$ inclusive, thus ensuring that $Q_{i-a}'$ holds. Thus $E_i'$ holds, contradicting the tentative assumption, which is therefore untenable. Therefore (56) is proved.

Suppose for example that $b=1$ and $c=2$. For $n > a+c+1 = a+3$, eq (57) yields

$$X_n Z_n = \left(\prod_{i=0}^{a} p_{n-i}\right) q_{n-a-1} q_{n-a-2} Y_{n-a-3}. \tag{58}$$

On the other hand, if $n > 2a+3$ then the last equation of section 4, with $n$ replaced by $n-1$, yields

$$\begin{aligned} X_n = Y_{n-1} - Y_n = q_{n-a}\left(\prod_{j=0}^{a-1} p_{n-j}\right) Y_{n-a-1} - \left(\prod_{j=0}^{a-1} p_{n-j}\right) q_{n-a} q_{n-a-1} Y_{n-a-2} \\ + \left(\prod_{j=0}^{a} p_{n-j}\right) q_{n-a-1} q_{n-a-2} Y_{n-a-3}. \end{aligned}$$

Comparison of this result with eq (58) shows that

$$q_{n-a}\left(\prod_{j=0}^{a-1} p_{n-j}\right) Y_{n-a-1} - \left(\prod_{j=0}^{a-1} p_{n-j}\right) q_{n-a}q_{n-a-1}Y_{n-a-2} = (1-Z_n)X_n,$$

and thus that

$$(1-Z_n)/Z_n = q_{n-a}(Y_{n-a-1} - q_{n-a-1}Y_{n-a-2})/p_{n-a}q_{n-a-1}q_{n-a-2}Y_{n-a-3}. \tag{59}$$

If $n>3a+3$, however, then eq (27), with $n$ replaced by $n-a-1$, yields (for $a>1$)

$$Y_{n-a-1} - q_{n-a-1}Y_{n-a-2} = p_{n-a-1}q_{n-a-2} + \sum_{k=2}^{a-1} q_{n-a-1-k}\left(\prod_{j=0}^{k-1} p_{n-a-1-j}\right) Y_{n-a-k-2} + \left(\prod_{j=0}^{a-1} p_{n-a-j-1}\right) q_{n-2a-1}q_{n-2a-2}Y_{n-2a-3}. \tag{60}$$

On the one hand, eq (60) shows that

$$Y_{n-a} - q_{n-a-1}Y_{n-a-2} \geq p_{n-a-1}q_{n-a-2}Y_{n-a-3}$$

so that eq (59) yields

$$(1-Z_n)/Z_n \geq q_{n-a}p_{n-a-1}/p_{n-a}q_{n-a-1}. \tag{61}$$

This shows, e.g., that if the sequence $\{p_n\}$ is nonincreasing for all $n>n_0>2a+3$, then $Z_n \leq 1/2$ for all $n>n_0+a$. A bit less crudely, since the sequence $\{Y_n\}$ is nonincreasing by its definition, eq (60) yields

$$Y_{n-a} - q_{n-a-1}Y_{n-a-2} \geq p_{n-a-1}Y_{n-a-3}\left\{ q_{n-a-2} + \sum_{k=2}^{a-1} q_{n-a-1-k}\left(\prod_{j=1}^{k-1} p_{n-a-1-j}\right) + \left(\prod_{j=1}^{a-1} p_{n-a-j-1}\right) q_{n-2a-1}q_{n-2a-2} \right\},$$

from which a more precise lower bound on $(1-Z_n)/Z_n$ can be derived *via* eq (59). On the other hand, since $Y_m \geq q_m Y_{m-1}$, we have

$$Y_{n-a-k-2} \leq Y_{n-a-3} \Big/ \prod_{i=3}^{k+1} q_{n-a-i},$$

$$Y_{n-2a-3} \leq Y_{n-a-3} \Big/ \prod_{i=3}^{a+2} q_{n-a-i},$$

which when substituted into eq (60) yield an inequality that can be combined with eq (59) to obtain a crude *upper* bound on $(1-Z_n)/Z_n$.

If $a=1$ (as well as $b=1$ and $c=2$) then eq (27) yields

$$Y_{n-2} - q_{n-2}Y_{n-3} = p_{n-2}q_{n-3}q_{n-4}Y_{n-5},$$

so that eq (59) becomes

$$(1-Z_n)/Z_n = q_{n-1}p_{n-2}q_{n-4}Y_{n-5}/p_{n-1}q_{n-2}Y_{n-4}. \tag{62}$$

Since $q_{n-4}Y_{n-5} \leq Y_{n-4} \leq Y_{n-5}$, we have

$$q_{n-1}p_{n-2}q_{n-4}/p_{n-1}q_{n-2} \leq (1-Z_n)/Z_n \leq q_{n-1}p_{n-2}/p_{n-1}q_{n-2}. \tag{63}$$

In particular, if $\{p_n\}$ is ultimately nondecreasing then ultimately $Z_n \geq 1/2$.

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# Two Theorems on Matrices*

Morris Newman

(May 16, 1962)

Generalizations of theorems important in the iterative solution of systems of linear equations are given, together with a lemma on the solution of a certain matrix equation.

## References

In this note we generalize two theorems on matrices which are of importance in certain iterative schemes for the solution of systems of linear equations (see [1] and [3] for relevant references).[1] In addition we prove a lemma on the solution of certain matrix equations. Although the proofs are simple, the results are of interest and find application in computations with matrices. However, we forego the discussion of these applications here.

In what follows $F$ and $M$ are $n \times n$ matrices and "*" denotes "conjugate transpose." We set

$$R=F+FM,\ S=F^*-FM \tag{1}$$

and assume that $R$ and $S$ are hermitian. We write p.d. for positive definite, n.s.d. for negative semi-definite, etc. The theorems we wish to prove follow.

THEOREM 1. *If* R *and* S *are both* p.d. *or both* n.d. *then every eigenvalue of* M *is of modulus less than* 1. *If* R *is* p.d. *and* S n.d. *or if* R *is* n.d. *and* S p.d., *then every eigenvalue of* M *is of modulus greater than* 1.

THEOREM 2. *If every eigenvalue of* M *is of modulus less than* 1, *then* R *is* p.d. *if* S *is* p.d. *and* n.d. *if* S *is* n.d. *If every eigenvalue of* M *is of modulus* $>1$, *then* R *is* p.d. *if* S *is* n.d. *and* n.d. *if* S *is* p.d.

Before giving the proofs of these theorems we state two lemmas. The first is well-known and we omit the proof. The proof of the second is given in detail.

Let $A=(a_{ij})$ be an arbitrary $n\times n$ complex matrix. If $B=(b_{ij})$ is a non-negative matrix such that $|a_{ij}| \leq b_{ij}$, we shall write $A<<B$. It is trivial to verify that if $A_1<<B_1$, $A_2<<B_2$ then $A_1+A_2<<B_1+B_2$, $A_1A_2<<B_1B_2$. Further, let $M(A)$ denote the absolute value of an eigenvalue of largest modulus of $A$.

LEMMA 1. *Let* A *be an arbitrary* n×n *complex matrix. Then there is a fixed non-negative matrix* B *such that for every integer* r≥0,

$$A^r<<r^{n-1}M(A)^rB. \tag{2}$$

This lemma is well-known and a proof can be given using the Jordan normal form of the matrix $A$. The term $r^{n-1}$ can be replaced by $r^{m-1}$, where $m$ is the order of a largest Jordan block of $A$. See also the paper [2] by A. Ostrowski.

LEMMA 2. *Let* A,B,C *be* n×n *complex matrices and suppose that*

$$\rho=M(A)M(B)<1. \tag{3}$$

*Then the matrix equation*

$$X-AXB=C \tag{4}$$

*has a unique solution* X *given by the infinite series*

$$X=\sum_{r=0}^{\infty}A^rCB^r=C+ACB+A^2CB^2+\ldots. \tag{5}$$

PROOF. Set

$$X_0=C,\ X_{r+1}=AX_rB+C,\ r\geq 0, \tag{6}$$

so that

$$X_r=C+ACB+\ldots+A^rCB^r.$$

If $r$ and $p$ are non-negative integers then it is easy to verify that

$$X_{r+p}-X_{r-1}=A^rX_pB^r.$$

By lemma 1 there are non-negative matrices $U$, $V$, $W$, independent of $r$ such that

$$A^r<<r^{n-1}M(A)^rU,\ C<<V,\ B^r<<r^{n-1}M(B)^rW.$$

Set $T=UVW$. Then

$$X_p=\sum_{r=0}^{p}A^rCB^r<<\left(\sum_{r=0}^{p}r^{2n-2}\rho^r\right)T,$$

where $\rho$ is defined by (3). Since $0\leq\rho<1$, this implies that $\sum_{r=0}^{\infty}r^{2n-2}\rho^r$ converges, and that

$$X_p<<\left(\sum_{r=0}^{\infty}r^{2n-2}\rho^r\right)T.$$

Hence lemma 1 implies that there is a non-negative matrix $K$ independent of $r$ and $p$ such that

$$X_{r+p}-X_{r-1}<<r^{2n-2}\rho^rK. \tag{7}$$

From (3) and (7) it follows easily that the sequence $\{X_r\}$ is a Cauchy sequence converging to a limit $X$, and (6) implies that $X=AXB+C$ and that $X$ satisfies (5). To show that the solution $X$ is

*The preparation of this paper was supported by the Office of Naval Research.
[1] Figures in brackets indicate the literature references at the end of this paper.

unique, let $Y$ be any other solution. Then $Y-X=A(Y-X)B$ and by iteration, $Y-X=A^r(Y-X)B^r$ for every non-negative integer $r$. Hence there is a non-negative matrix $L$ independent of $r$ such that $Y-X<<r^{2n-2}\rho^r L$. Letting $r$ go to infinity, (3) implies that $Y=X$. The proof of the lemma is complete. We go on to the proof of theorem 1.

Let $\lambda$ be an eigenvalue of $M$, $v$ a corresponding eigenvector, and set $f=v^*Fv$, $r=v^*Rv$, $s=v^*Sv$. Then (1) implies that

$$r=f+\lambda f,\ s=\bar{f}-\lambda f. \tag{8}$$

Assume that $R$ and $S$ are definite matrices. Then $\lambda\neq -1$ and we can eliminate $f$ from (8). We obtain

$$\frac{r}{s}=\frac{1-|\lambda|^2}{|1+\lambda|^2} \tag{9}$$

Theorem 1 is implied by (9) since $r/s>0$ if $R$ and $S$ are both p.d. or both n.d., and $r/s<0$ if $R$ is p.d. and $S$ n.d. or $R$ is n.d. and $S$ p.d.

We now turn to the proof of theorem 2. If we eliminate $F$ from (1) (remembering that $R$ and $S$ are hermitian) we obtain

$$(I+M^*)S(I+M)=R-M^*RM. \tag{10}$$

If we assume that every eigenvalue of $M$ is of modulus less than 1 then lemma 2 applies and we obtain

$$R=\sum_{r=0}^{\infty}(M^*)^r(I+M^*)S(I+M)M^r. \tag{11}$$

Since the sum of any number of p.s.d. matrices is p.s.d. and the sum of any number of n.s.d. matrices n.s.d., the matrix $R$ is p.s.d. if $S$ is p.s.d. and n.s.d. if $S$ is n.s.d. But the first term of the sum (11) is p.d. if $S$ is p.d. and n.d. if $S$ is n.d. Thus the first part of theorem 2 is proved.

Assume now that every eigenvalue of $M$ is of modulus greater than 1. Then $M$ is nonsingular and every eigenvalue of $N=M^{-1}$ is of modulus less than 1. Equation (10) may be written

$$(I+N^*)S(I+N)=N^*RN-N. \tag{12}$$

Once again lemma 2 applies and we obtain

$$R=-\sum_{r=0}^{\infty}(N^*)^r(I+N^*)S(I+N)N^r, \tag{13}$$

from which the second part of theorem 2 follows. This concludes the proof of theorem 2.

For related theorems, see the paper [4] by P. Stein. Lemma 2 of our paper can be used to provide a simplified proof of the theorems given by Stein in [4] and indeed to generalize these theorems.

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# Mill's Ratio for Multivariate Normal Distributions[1]

I. Richard Savage[2]

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Two easily applied inequalities are given for the "tail probabilities" of multivariate normal distributions.

A basic integral arising in statistics is of the form:

$$F(\mathbf{a},\mathbf{M})=\frac{|\mathbf{M}|^{1/2}}{(2\pi)^{n/2}}\int_0^\infty \cdots \int_0^\infty \exp\left[-\frac{1}{2}(\mathbf{x}+\mathbf{a})\mathbf{M}(\mathbf{x}+\mathbf{a})'\right] dx_1 \ldots dx_n \tag{1}$$

where $\mathbf{a}$ and $\mathbf{x}$ are $n$ component row vectors, $\mathbf{M}$ is an $n\times n$ positive definite symmetric matrix with determinant $|\mathbf{M}|$ and "′" means transpose. In particular (1) arises in evaluating $Pr(\mathbf{X}\geq\mathbf{b})$ where $\mathbf{X}$ is a random variable with a multivariate normal distribution, and $\mathbf{b}$ is a vector of constants. In this note inequalities (upper and lower bounds) are presented for $F(a,\mathbf{M})$.

Let

$$f(\mathbf{x},\mathbf{M})=\frac{|\mathbf{M}|^{1/2}}{(2\pi)^{n/2}}\exp\left(-\frac{1}{2}\mathbf{x}\mathbf{M}\mathbf{x}'\right),$$

then

$$F(\mathbf{a},\mathbf{M})=f(\mathbf{a},\mathbf{M})\int_0^\infty \cdots \int_0^\infty \exp\left(-\mathbf{a}\mathbf{M}\mathbf{x}'\right)\times\exp\left(-\frac{1}{2}\mathbf{x}\mathbf{M}\mathbf{x}'\right) dx_1 \ldots dx_n. \tag{2}$$

Assume throughout the following,

$$\boldsymbol{\Delta}=\mathbf{a}\mathbf{M}>0, \text{ i.e., } \Delta_i=\sum_{j=1}^{n} a_j m_{ji}>0 \text{ for } i=1, \ldots, n. \tag{3}$$

The following elementary inequalities are required

$$\sum_{k=0}^{2r+1}(-c)^k/k!<e^{-c}<\sum_{k=0}^{2r}(-c)^k/k! \text{ for } r=0, 1, \ldots. \tag{4}$$

and $c>0$. In particular, (4) will be used with $c=\frac{1}{2}\mathbf{x}\mathbf{M}\mathbf{x}'$, the last quantity being positive whenever $\mathbf{x}\neq 0$, i.e., at least one $x_i\neq 0$, since $\mathbf{M}$ is positive definite.

Now using the right-hand side of (4) with $r=0$ in (2) one obtains

$$F(\mathbf{a},\mathbf{M})<f(\mathbf{a},\mathbf{M})\int_0^\infty \cdots \int_0^\infty \exp\left(-\sum_{i=1}^{n}\Delta_i x_i\right) dx_1, \ldots, dx_n. \tag{5}$$

After evaluating the integral in (5) one has

$$F(\mathbf{a},\mathbf{M})<f(\mathbf{a},\mathbf{M})\left(\prod_{i=1}^{n}\Delta_i\right)^{-1}. \tag{I}$$

Now using the left-hand side of (4) with $r=0$ in (2) one obtains

$$F(\mathbf{a},\mathbf{M})>f(\mathbf{a},\mathbf{M})\int_0^\infty \cdots \int_0^\infty \left(1-\frac{1}{2}\sum_1^n\sum_1^n x_i m_{ij} x_j\right)\times \exp\left(-\sum_{i=1}^{n}\Delta_i x_i\right) dx_1, \ldots, dx_n. \tag{6}$$

[1] Work supported in part by the Office of Naval Research.

[2] Present address: University of Minnesota, Minneapolis, Minn.

After evaluating the integral in (6) one obtains

$$F(\mathbf{a},\mathbf{M})>f(\mathbf{a},\mathbf{M})\left(\prod_{i=1}^{n}\Delta_i\right)^{-1}\left[1-\frac{1}{2}\sum_{1}^{n}\sum_{1}^{n}m_{ij}(1+\delta_{ij})/(\Delta_i\Delta_j)\right]. \tag{II}$$

where $\delta_{ij}=1$ if $i=j$ and $\delta_{ij}=0$ if $i\neq j$.

Inequalities similar to (I) and (II) can be obtained by using $r>0$ in (4). These inequalities would be complicated notationally. In particular, however, Feller [1, pp. 166, 179][3] presents these inequalities (for general $r$) when $n=1$ and $m_{11}=1$. Pólya [2] has considered the case of $r>0$ for the integrand in (1) for a different region of integration. Pólya's sequence of inequalities has the desirable property of improving as $r$ increases until the sharpest form is obtained and then of becoming successively weaker. Ruben [3] has considered the special case where $m_{ii}=1$ for $i=1,\ldots,n$ and $m_{ij}=m$ for $i\neq j$. His results appear to be complicated.

If the case considered by Ruben is specialized in the following manner: $n=2$ and $a_1=a_2=a$, then one obtains from using (4) in (2) the following

$$F(\mathbf{a},\mathbf{M})\lessgtr f(\mathbf{a},\mathbf{M})\sum_{i=0}^{i^*}\Delta^2(-2\Delta^2)^{-i}\left[\sum^{(i)}\frac{(2u+v)!(2w+v)!(2m)^v}{u!v!w!}\right], \tag{III}$$

where $>$ ($<$) if $i^*$ is odd (even), $\Delta=1+m$, and $\Sigma^{(i)}$ is over all $u,v,w\geq 0$ such that $u+v+w=i$. It is not clear that the inequalities in (III) have the desirable feature of the Pólya bounds.

*Example 1.* Assume $\mathbf{X}$ has a bivariate normal distribution with mean vector zero, and variance covariance matrix $\mathbf{M}^{-1}=\begin{vmatrix}1 & 1/2\\ 1/2 & 1\end{vmatrix}$. Approximate $Pr(\mathbf{X}\geq(3,3))$. The integral expression for the desired probability is

$$\int_3^\infty\int_3^\infty f(\mathbf{x},\mathbf{M})dx_1dx_2.$$

In this integral make the following change of variables $y_1=x_1-3$ and $y_2=x_2-3$. The integral then becomes $F(\mathbf{a},\mathbf{M})$ where $\mathbf{a}=(3,3)$, $\mathbf{M}=\begin{vmatrix}4/3 & -2/3\\ -2/3 & 4/3\end{vmatrix}$, and $|\mathbf{M}|=4/3$. Note $\Delta_1=\Delta_2=2$. Now using (I) one obtains

$$Pr(\mathbf{X}\geq(3,3))<\frac{(4/3)^{1/2}e^{-6}}{8\pi}\doteq 0.000114,$$

and from (II) one obtains

$$Pr(\mathbf{X}\geq(3,3))>0.000114\left(1-\frac{16/3-4/3}{8}\right)\doteq 0.000057.$$

From [5] one finds the correct value to be 0.000082.

In this example the ratio of the upper to the lower bound is 2. If one considers the same problem, with positive correlations of 1/2 replaced by $-1/2$ the corresponding ratio is 54/49. Since the probabilities are smaller with negative than with positive correlations, this is not a surprising result.

*Example 2.* Assume $\mathbf{M}=\begin{vmatrix}4/3 & 2/3\\ 2/3 & 4/3\end{vmatrix}$ so that $|\mathbf{M}|=4/3$, also assume $\mathbf{a}=(2,2)$ so that $\boldsymbol{\Delta}=(4,4)$ then

$$0.0000039>F(\mathbf{a},\mathbf{M})>0.0000031.$$

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[3] Figures in brackets indicate the literature references at the end of this paper.

To compare the accuracy of the results for the bivariate case with the univariate case, consider the following univariate example where the true value is about the same as in the above bivariate example. Let $n=1$, $m_{11}=1$, and $a=4.5$, then one obtains

$$0.00000355>F(4.5,\,1)>0.00000337.$$

The correct value, from [4], is 0.000003398.

In the present notation, the general result for the univariate case given by Feller [1, p. 179] is

$$F(a,1)\gtrless a^{-1}f(a,1)\sum_{r=0}^{R}\frac{(-1)^r(2r)!}{2^r r!a^{2r}},$$

where $a>0$ and $>(<)$ if $R$ is odd (even).

*Example 3.* If $m_{ii}=1$ for $i=1,\ldots,n$ and $m_{ij}=m$ for $i\neq j$, and $\mathbf{a}=(a,\ldots,a)$, then $|\mathbf{M}|=(1-m)^{n-1}[1+(n-1)m]$ and $\Delta_i=a[1+(n-1)m]$. (To preserve the positive definite character of $\mathbf{M}$ it is necessary that $1+(n-1)m>0$, i.e., $m>-(n-1)^{-1}$ and $m<1$.) Now using (I) and (II), one obtains

$$F(\mathbf{a},\mathbf{M})<|\mathbf{M}|^{1/2}e^{-na\Delta/2}(2\pi)^{-n/2}\Delta^{-n}$$

and

$$F(\mathbf{a},\mathbf{M})>|\mathbf{M}|^{1/2}e^{-na\Delta/2}(2\pi)^{-n/2}\Delta^{-n}\left[1-\frac{n(1+\Delta)}{2\Delta^2}\right],$$

where $\Delta=1+(n-1)m$. Consider the case with $n=3$, $m=1/3$ and $a=5(3/2)^{1/2}$. (This numerical example corresponds to finding the probability that a trivariate normal variable with mean vector zero, and covariance matrix having 1's on the diagonal and 1/2's off the diagonal has each component larger than 5.) In this case the ratio of the upper bound to the lower bound of $F(\mathbf{a},\mathbf{M})$ is 25/13. If one considers $a=9(3/2)^{1/2}$ the corresponding ratio is 27/23.

With the previous notation and restrictions, consider a new integral, $G(\mathbf{a},\mathbf{M})$ obtained from modifying $F(\mathbf{a},\mathbf{M})$ by replacing the upper limits of integration by ones, i.e.,

$$G(\mathbf{a},\mathbf{M})=\int_0^1\cdots\int_0^1 f(\mathbf{x}+\mathbf{a},\mathbf{M})\,dx_1\ldots dx_n. \tag{7}$$

Then evaluating the integral analogous to (5), one obtains

$$G(\mathbf{a},\mathbf{M})<f(\mathbf{a},\mathbf{M})\left(\prod_{i=1}^{n}[1-e^{-\Delta_i}]/\Delta_i\right). \tag{IV}$$

From the analogue of (6), one obtains

$$G(\mathbf{a},\mathbf{M})>f(\mathbf{a},\mathbf{M})\left[\prod_{i=1}^{n}(1-e^{-\Delta_i})/\Delta_i\right]\left\{1-\frac{1}{2}\left[\sum_{i=1}^{n}\frac{m_{ii}}{\Delta_i}\left(1+\frac{1-(\Delta_i+1)^2e^{-\Delta_i}}{1-e^{-\Delta_i}}\right)+\sum\sum_{i\neq j}\frac{m_{ij}}{\Delta_i\Delta_j}\left(1-\frac{\Delta_i e^{-\Delta_i}}{1-e^{-\Delta_i}}\right)\left(1-\frac{\Delta_j e^{-\Delta_j}}{1-e^{-\Delta_j}}\right)\right]\right\}. \tag{V}$$

Results (IV) and (V) were obtained by Professor Olkin.

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A first draft of this paper was prepared in 1954 while the author was at the National Bureau of Standards. That work was stimulated by Dr. Benjamin Epstein in conversations on the distribution of extreme values for dependent variables. Professor Ingram Olkin suggested changes in the original draft which have been incorporated at this time. During the summer of 1961, under a National Science Foundation Undergraduate Research Program, Thomas Kieran did some additional numerical analysis for the manuscript.

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# Angle as a Fourth Fundamental Quantity

Jacques E. Romain*

(April 18, 1962)

The advantage of considering angle as a fourth fundamental quantity of geometry and physics is stressed, and an alternative approach is suggested to introduce an angular dimension in such a way that the physical laws, in any form, are dimensionally homogeneous. A few examples are described to show how some equations of mathematics and mechanics should be revised to put the new point of view into practice.

## 1. Introduction

Chester H. Page[1] recently wrote in this Journal a most interesting study about "physical entities and their mathematical representations." One of the ideas in that paper, which will be of interest here, is that advantage can be gained from considering angle as an additional fundamental quantity. Dr. Page rightly points out that, since the measure of an angle depends on the chosen angle unit, it would be natural not to consider it a dimensionless quantity. That point of view could hardly be overstressed, and the present writer has the unpleasant feeling of an artificial trick about the generally accepted rule that angle is a dimensionless quantity, but may only be measured in radians if trouble is to be avoided.

Moreover, ignoring the dimension of angle deprives dimensional analysis of a part of its fruitfulness. Although the number of fundamental quantities in a dimensional theory is not assigned by any logical prescription, it is well known[2] that the optimum number of fundamental quantities is the smallest number that is enough to express every physical quantity in the theory in terms of properly chosen fundamental quantities, without arbitrarily assigning a dimensionless status to any physical constant. In view of these arguments, it seems rather surprising that earlier attempts[3, 4, 5, 6] did not seem to meet any widespread audience.

This paper describes an alternative approach to the introduction of an angular dimension. It must not be construed as polemics against Page's approach: actually both systems are equally consistent. However, let it be noted that the present point of view does away with the unappealing feature, in Page's system, that Euler's formula

$$e^{ix}=\cos x+i\sin x \qquad (i^2=-1)$$

is dimensionally nonhomogeneous, and that therefore dimensional homogeneity is not preserved when the complex exponential formalism is used in physics instead of trigonometric functions. Of course, the difficulty can be disposed of by stating that "this is an illustration of the fact that equations used in physics need not be dimensionally homogeneous when terms arise from the use of an artifice. The function $e^{i\theta}$ does not actually occur in physics; it is introduced with the convention that its real part alone (or its imaginary part alone) represents a variable of interest. Nonartificial exponentials, such as $e^{-W/kT}$, have numeric exponents." (See footnote 1.) But it seems simpler to define the angular dimension so that Euler's formula is dimensionally homogeneous; that will be done presently. (However, the possibility of reconciling Page's scheme with the requirement of homogeneity will be shown at the end of section 3.) Moreover, the present treatment seems to allow a more straightforward extension to solid angles, taken as another distinct fundamental quantity, on the same footing with plane angle, by simply substituting "steradian" for "radian" and "area" for "length" in eq (1a).

On the other hand, this attempt misses the pleasant feature, in Page's work, that torque and energy acquire different dimensions (as do action and angular momentum, or areal velocity and kinematic viscosity coefficient). But there really is no reason, in the theory of dimensions, to demand that no two different physical quantities should have the same dimensional formula; and it seems hardly possible to devise a dimensional system in which no such duplication would happen anywhere: for instance, even in Page's system, a frequency and a velocity gradient have the same dimensions.

## 2. Basic Definitions

The appearance of angles in an equation can often be traced to the relation

$$s=\theta R \qquad (1)$$

between an arc $s$ of a circle, its radius $R$ and the central angle $\theta$ measured in radians. The necessity of dimensional homogeneity of both sides of eq (1) (in which $s$ and $R$ are lengths) seems to imply that $\theta$ is dimensionless.

*Present address, General Dynamics, Fort Worth, Texas.

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It has been tried (see footnote 6) to avoid contradiction by taking into consideration two kinds of "angles": the "geometrical angle" which is the usual one, a quantity related to the portion of the plane contained between two concurrent straight lines, and the "analytical angle" which is the $\theta$ to be found in eq (1). But the so-called "analytical angle" is no new notion indeed: it is plainly the measure of the angle in radians. This can be seen immediately from eq (1), which originates in a theorem stating that in a circle two arcs are proportional to their central angles. When one of the arcs is equal to the radius its central angle is one radian, and the proportion becomes

$$s=(\theta/1\text{ rad})R, \tag{1a}$$

which is the correct form of eq (1), with $\theta$ a true "geometrical" angle.

How is it that one should think himself obliged to particularize the measures of angles under the name of "analytical angles," while not feeling any need whatsoever of "analytical lengths," "analytical masses" or "analytical time"? The explanation, and the reason why mechanics could be developed without recognizing the fundamental dimension of angle, is that angles often appear in mechanics through their measure in radians, which indeed can be seen to be derived from eq (1a). Something similar happens in other chapters of physics: for instance it is well-known that the dimensional analysis in photometry is less complete if the further fundamental dimension of solid angle is not taken into account; actually what is here being said of angles can be extended to solid angles.

The basic angular quantity will be taken here to mean the ordinary "geometrical" angle, certainly the most primary notion in any study of angles. Trigonometric functions will be defined in the elementary way as ratios of lengths, i.e., dimensionless quantities. This is simpler than starting from power expansions, and has the advantage of putting *sin* and *cos* on an equal footing.

Since the denominator (1 rad) will often appear, I shall write for the sake of brevity, for any quantity $Q$:

$$Q/(1\text{ rad})=\langle Q\rangle. \tag{2}$$

This should not be mistaken for the dimensionless measure of $Q$: it is so only for angles. (When $Q$ is an angle $\theta$, $\langle\theta\rangle$ is simply the "analytical" alternative to the "geometrical" angle $\theta$.) Thus eq (1a) will be written

$$s=\langle\theta\rangle R. \tag{3}$$

## 3. Angular Dimension in Mathematics

Restrictions are met in mathematics about angles, at least on two occasions: these are eq (1), and the following:

$$\lim_{\theta\to 0}(\sin\theta)/\theta=1, \tag{4}$$

both of which are only true when $\theta$ is measured in radians. Equation (1) has just been dealt with, and its general form (3) has been obtained. The same can be done to eq (4), which is a direct daughter of eq (1). The reader should turn back to the elementary demonstration of eq (4); the general scheme is: (1) show that the length of an arc $\theta$ of a circle lies between those of two line segments which are proportional to $\sin\theta$ and $\tan\theta$; (2) divide by the radius $R$: the two segments yield $\sin\theta$ and $\tan\theta$, while the arc becomes $s/R$, or $\langle\theta\rangle$ on account of eq (3). Thus, the true form of eq (4), valid for any unit of angle, is

$$\lim_{\theta\to 0}(\sin\theta)/\langle\theta\rangle=1. \tag{5}$$

A similar equation would be obtained with $\tan\theta$ instead of $\sin\theta$.

The equations showing how to replace the sine and tangent of a small angle by the angle itself follow in the correct form

$$\sin\theta\approx\langle\theta\rangle;\ \tan\theta\approx\langle\theta\rangle. \tag{6}$$

As a consequence, every equation derived from eq (3) or (5) should be recalculated. The details of the calculations are left to the reader.

Here are a few results:

(a) Derivatives of trigonometric functions:

$$d(\sin x)/dx=(1/1\text{ rad})\cos x, \tag{7}$$

or

$$d(\sin x)=\cos x\, d\langle x\rangle, \tag{8}$$

and analogous expressions for the derivatives of $\cos x$, $\tan x$, . . . . An application of eq (7) which will be of further use is

$$d(\sin\omega t)/dt=\langle\omega\rangle\cos\omega t. \tag{9}$$

(b) By comparing the power expansions of $e^{ix}$ to those of $\cos x$, $\sin x$ derived from eq (7), namely

$$\cos x=1-\frac{1}{2}\langle x\rangle^2+(1/4!)\langle x\rangle^4-\ldots,$$

$$\sin x=\langle x\rangle-(1/3!)\langle x\rangle^3+\ldots,$$

one readily obtains for Euler's equation the dimensionally homogeneous form

$$e^{i\langle x\rangle}=\cos x+i\sin x, \tag{10}$$

which will be used in the form

$$e^{i\langle\omega\rangle t}=\cos\omega t+i\sin\omega t.$$

From eq (10) follows

$$\cosh i\langle x\rangle=\cos x;\ \sinh i\langle x\rangle=i\sin x. \tag{11}$$

(c) When remembering eq (7) and computing again the following integral, one gets

$$\int(a^2-y^2)^{-1/2}dy=(1/1\text{ rad})\sin^{-1}(y/a)+C$$
$$=\langle\sin^{-1}(y/a)\rangle+C,$$

and similar expressions for $\cos^{-1}$ and $\tan^{-1}$.

It can be noticed that all the above equations are homogeneous with respect to dimensions ($x$ is an angle, $t$ the time, $y$ and $a$ any quantities of the same dimension; $\omega$ has the dimension $\Theta T^{-1}$, if $\Theta$ denotes the angular dimension): the arguments of trigonometric functions are angles, but those functions themselves are pure dimensionless numbers, as well as the arguments of hyperbolic functions and the exponents. With those equations in mind no difficulty can be experienced in setting up the equations of mechanics.

Before switching to mechanical equations, let it be pointed out that homogeneity can easily be restored in Page's scheme for the equation

$$e^{ix}=\cos x+i\sin x,$$

and more generally for the polar expression of any complex number, wherein the same problem occurs.

What needs be done is simply assigning the dimension of angle to the complex "$i$". Since $[\theta]^2=[1]$ (Page's notation), there is no conflict with

$$i^2=-1$$

as a quantity equation; and as $\sin x$, in that scheme, also has the dimension of angle, that would make both the exponent $ix$ and the product $i\sin x$ dimensionless.

The unusual feature is that in any complex number $a+bi$, $a$ is a numeric but $b$ has the dimension of angle (or more generally $a$ has the dimensions of the modulus and $b$ those of modulus times angle). The appearance of an angular dimension in complex number representation would not seem incongruous if one remembers that the natural geometric representation of complex numbers is in a plane.

But the feature just alluded to implies a further consequence. If homogeneity is to be preserved throughout, the imaginary part $b$ of *any* complex number $a+bi$ must have the dimension of angle, even when $b=1$. This implies introducing some arbitrary "unit of imaginary part," say $u$, with the dimension of angle, by means of which the complex number $(a+i)$ can be written $(a+ui)$. It is then obvious that the use of $u$ is completely equivalent to putting

$$j=ui$$

($j$ dimensionless) and writing

$$a+bi=a+(b/u)j.$$

Now the dimensionless $b/u$ is exactly the expression defined by eq (2). In other words, in the suggested "homogenized Page's formalism," the equations involving complex numbers go over into a form equivalent to that of the present approach.

The comparison between Page's scheme and the present proposal can thus be summarized as follows:

In Page's scheme the relations

$$d(\sin x)/dx=\cos x;\ e^{ix}=\cos x+i\sin x$$

are quantity equations, whereas in the present proposal eqs (8) and (10) include explicit reference to the radian. But this advantage of Page's scheme is, to this writer's feeling, canceled by the necessity of either introducing $u$ in the complex numbers or retaining inhomogeneous equations wherever complex numbers are involved in the polar form.

The only irreducible difference between the two approaches boils down to the fact that in Page's scheme the dimension of angle is singled out by the requirement that $[\theta]^2=[1]$, while all four dimensions, as such, are put on an equal footing in the present approach. (In fact, however, the explicit reference to the radian, or to $u$, also singles out the angular dimension in both systems.) Admittedly, this is essentially a matter of taste.

## 4. Angular Dimension in Kinematics

The basic appearance of angles in kinematics is through angular velocity. The mutual dependence of linear and angular velocities in a circular motion is obtained by differentiating eq (3) with respect to time, and putting $v$ for $ds/dt$ and $\omega$ for $d\theta/dt$:

$$v=\langle\omega\rangle R. \tag{12}$$

This equation is dimensionally homogeneous, for the dimensional formula of $\omega$ is $\Theta T^{-1}$; thus that of $\langle\omega\rangle$ is $T^{-1}$.

Let us now investigate a few consequences of eq (12). From eq (12) the dimensionally correct expressions can be deduced for the rotational kinetic energy of a solid:

$$T=\sum\frac{1}{2}mv^2=\frac{1}{2}I\langle\omega\rangle^2,$$

for the centrifugal force on a particle:

$$F_c=m\langle\omega\rangle^2R,$$

and for the law of areas:

$$R^2d\langle\theta\rangle/dt=C.$$

The angular velocity vector $\mathbf{\Omega}$ of a rotating solid is defined by the equation

$$\mathbf{v}=\mathbf{\Omega}\times\mathbf{r}. \tag{13}$$

It is usually considered to have a length equal to the value of the angular velocity $\omega$. The dimensional homogeneity of eq (13) (where the cross-product symbol is dimensionless) shows that the true relation, when angle is considered a fundamental quantity, is

$$\Omega=\langle\omega\rangle, \tag{14}$$

with the dimensional formula $T^{-1}$. This dimensional formula is found to agree with the particular case of the vortex vector of a fluid:

$$\mathbf{\Omega}=\frac{1}{2}\nabla\times\mathbf{v},$$

in which no angular dimension appears.

Binet's formula expressing the radial acceleration of a point obeying the law of areas reads

$$j_r=-(A^2/r^2)\cdot[d^2(1/r)/d<\theta>^2+1/r],$$

where $A$ is the constant of area.

And, finally, the radius of curvature $R$ of a curve in a point is defined, in terms of the infinitesimal arc $ds$ and the corresponding angular displacement $d\theta$ of the tangent, by

$$R=ds/d<\theta>, \tag{15}$$

which is a direct application of eq (3).

## 5. Angular Dimension in Dynamics

The appearance of angles in dynamics may derive from many of the above equations:

From eq (6) comes the proper form of the equation yielding the shear angle:

$$<\psi>=2T(1+\sigma)/E \tag{16}$$

(where $\sigma$ is Poisson's ratio and $E$ Young's modulus), and the physical expression of the nondiagonal components $\epsilon_{ij}$ of the infinitesimal strain tensor of a continuous medium in terms of the shear angles relevant to the directions $i$, $j$:

$$\epsilon_{ij}=<\psi_{ij}>.$$

Also from eq (6) follows the second-order approximation of the formula for the period of the circular pendulum with amplitude $\theta$:

$$\tau=2\pi(l/g)^{1/2}(1+<\theta>^2/16+\ldots).$$

Equation (9) leads to the expression of the equation for the harmonic motion of a point on a straight line under a central attractive force $-kr$ proportional to distance:

$$x=(v_0/<\omega>)\sin\omega t+x_0\cos\omega t, \tag{17}$$

with

$$<\omega>=(k/m)^{1/2}; \tag{18}$$

the period is

$$\tau=2\pi/<\omega>$$

and has the correct dimension of a time.

The analogous motion under a central repulsive force $+kr$ is deduced from eq (17) by changing $k$ into $-k$ in eq (17) and keeping eq (18) as a definition of $\omega$; it reads, thanks to eq (11):

$$x=x_0\cosh<\omega>t+(v_0/<\omega>)\sinh<\omega>t.$$

Equations (13), (14) show that the relation between the (normal) torque $\mathfrak{M}$ on the axis causing a gyroscope to precess, the moment of inertia $I$ about that axis, and the angular velocities of rotation and of precession is

$$\mathfrak{M}=<\omega><\omega'>I,$$

where $\mathfrak{M}$ has the conventional dimension.

Equation (15) is responsible for the formulation of the law of distribution of tension in a string constrained to lie on a surface with friction, in terms of the angle $\theta$ of rotation of the tangent:

$$T=T_0e^{f<\theta>}.$$

The same eq (15) would be involved in the study of the motion of a point missile in the air with friction.

The dimensional homogeneity shows that the relation between torque, angle of rotation, and the torsional constant in a twisted wire is

$$\mathfrak{M}=C<\theta>,$$

which can also be seen theoretically to proceed from eq (16). The expression for the torsional period follows with the correct dimension:

$$\tau=2\pi(I/C)^{1/2}=2\pi(I<\theta>/\mathfrak{M})^{1/2}.$$

## 6. Conclusion

The above survey does not claim to be anything like an exhaustive study of the introduction of an angular dimension in mechanics. It is just a collection of examples to show that no disadvantage can arise from its systematic use. Actually, this writer taught rational mechanics for five years with this system of dimensions without ever coming across any inconvenience.[7] On the contrary, he found the new point of view often illuminating since it allows an appreciable widening of the scope of dimensional analysis.

[7] J. E. Romain, Publications de l'Université de l'Etat, Elisabethville (Katanga), No. 3, 1962.

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# Invalidity of Meixner's Theorem in Irreversible Thermodynamics

R. E. Nettleton

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Meixner's theorem, on the preservation of the Onsager symmetry of phenomenological coefficients under transformations of forces and fluxes which leave the rate of entropy production invariant, is shown to be invalid under the restriction that all fluxes be even or odd under time-reversal and that the phenomenological equations be valid for very short times. This result extends an earlier proof of Coleman and Truesdell, whose proof does not impose these two restrictions. A new formulation of Meixner's theorem is thus shown to be required, and this is written down together with a statement of Onsager's theorem which is consistent with it.

## 1. Introduction

DeGroot has proposed[1] that the theory of irreversible processes should rest on a fundamental postulate which, from the manner in which it has been used by deGroot and others, can be summarized by the following statement:

STATEMENT I: Suppose $S$ is the entropy of a thermodynamic system and $\dot{S}$ the positive definite part[2] of its rate of change with respect to time $t$, and also that $X_a (a=1, \ldots, n)$, called "forces," are a set of linearly independent functions of the thermodynamic variables such that $\dot{S}$ can be written in the form[3]

$$\dot{S}=\sum_{a=1}^{n} J_a X_a \tag{1}$$

where $J_a$ is called the "flux conjugate to $X_a$." Then the statement

$$J_a=\sum_{b=1}^{n} L_{ab} X_b \tag{2}$$

implies, in the absence of a magnetic field, that

$$L_{ab}=L_{ba} \tag{3}$$

if[4] $X_a(t)=X_a(-t)$ and $X_b(t)=X_b(-t)$ or $X_a(t)=-X_a(-t)$ and $X_b(t)=-X_b(-t)$. In the first of these two cases, $X_a$ and $X_b$ are said to be *symmetric* with respect to time inversion, and in the second case *antisymmetric*. Also

$$L_{ab}=-L_{ba} \tag{4}$$

if $X_a$ or $X_b$ is symmetric and the other of the two antisymmetric.

Coleman and Truesdell[5] have shown that Statement I, the content of which is widely accepted as a valid statement of the Onsager-Casimir reciprocity theorem, goes beyond existing proofs of the theorem which assume that the forces $X_a$ are derivatives $\partial S/\partial y_a$ of $S$ with respect to a set of state variables, and that the flux $J_a$ conjugate to $X_a$ is equal to the time-derivative $\dot{y}_a$. These authors (see footnote 5) have pointed out that it is common to apply Statement I in cases where the $X_a$ are gradients of concentration or fluid velocity, which are types of forces for which it has never been proved in complete generality.[6] In fact, they have shown that, without additional restrictions on the forces $J_a$ and fluxes $X_a$ beside eq (1) and the requirement of linear independence, Statement I is indeed false. To establish its invalidity, it suffices to assume the existence of $n$ pairs of functions $X_a$, $J_a$ for which eqs (1 to 4) hold, and then to find $n$ new pairs $J_a^*$, $X_a^*$ which are linear combinations of the $J_a$ and $X_a$ such that eqs (1) and (2) hold when all quantities therein are starred, while eqs (3) and (4) cease to hold for the matrix $\mathfrak{L}^*=(L_{ab}^*)$ relating the $J_a^*$ to the $X_b^*$.

Such a transformation to new forces and fluxes violates a widely accepted statement, called "Meixner's theorem," which asserts that a transformation of this type cannot be found. This assertion goes beyond what is claimed by Meixner himself, who has defined[7] a particular class of linear transforma-

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[1] S. R. deGroot, Thermodynamics of Irreversible Processes, North-Holland Publishing Company, Amsterdam, 1951; see pp. 8, 216.

[2] In open systems, it is usually possible to express the total time rate of change of entropy as the sum of two terms, one of which reduces with the aid of the phenomenological laws of heat conduction, diffusion, and viscous dissipation, to a positive definite quadratic form. The other term is recognizable as an exchange of entropy with the surroundings which, in the theory of continuous systems [ref. 1, p. 98] appears as the divergence of an entropy flow vector.

[3] In the present statement no requirement is placed on the quantities $X_a$ other than that they be linearly independent and satisfy eq (1). It will be the purpose of the present paper to show that these requirements are inadequate for the truth of the statement and to establish the additional conditions which must be placed on the forces and on the phenomenological coefficients $L_{ab}$ if the statement is to be so formulated as to have general validity.

[4] $X_a((t)$ is a notation which refers to an ensemble average $\langle X_a(\mathbf{P},\mathbf{Q})\rangle$ of some function of the coordinates $\mathbf{Q}$ and momenta $\mathbf{P}$ of all the molecules in the system. The transformation $t\to -t$ means reversal of the signs of the momenta inside the angular bracket.

[5] B. D. Coleman and C. Truesdell, J. Chem. Phys. **33,** 28 (1960).

[6] Statement I has been shown to hold, by S. R. deGroot and P. Mazur, Phys. Rev. **94,** 218, 224 (1954), in special cases of heat conduction and diffusion, but this demonstration cannot be automatically extended to cases where gradients of temperature or concentration are coupled in the phenomenological equations with forces of the type $\partial S/\partial y_a$.

[7] J. Meixner and H. G. Reik, Handbuch der Physik, Vol. 3, part 2, Ed. by S. Flügge, Springer Verlag, Berlin, 1959; see pp. 433, 434.

tions from one set of fluxes $J_a$ and forces $X_a$ to another set $J_a^*$ and $X_a^*$ which have the property that if eqs (1 to 4) hold for the $J_a, X_a$, then these equations continue to hold when all quantities are starred. Transformations with this property are said to keep $\dot{S}$ and the Onsager symmetry *invariant* and are called *symmetry-conserving* transformations. From the existence of this class of transformations, it is commonly inferred, incorrectly, that all linear transformations which keep $\dot{S}$ invariant are also symmetry-conserving. The latter statement is the broad statement of "Meixner's theorem," which Meixner himself [compare ref. 7] realized could not be true in complete generality, although he did not discuss, as we shall do here, the various types of transformations which contradict it. Thus it has been left to Coleman and Truesdell (see footnote 5) to describe one set of transformations which keeps $\dot{S}$ invariant but is not symmetry-conserving, and which leads to counter-examples for Statement I.

These counter-examples (see footnote 5) are obtained by a transformation to new fluxes $J_a^*$, given by

$$J_a^* = J_a + \sum_b W_{ab} X_b$$

where $W_{ab} = -W_{ba}$ and it is assumed that any or all elements in the matrix $(W_{ab})$ are $\neq 0$. The antisymmetry of $(W_{ab})$ assures that if $X_a^* \equiv X_a$, then $\sum_a J_a^* X_a^* = \sum_a J_a X_a$. From this it readily follows (see footnote 5), without regard to whether the $X_a$ are all odd (antisymmetric) or all even (symmetric) or a mixture of the two, that the $J_a^*$ and $X_a^*$ can satisfy conditions (1) and (2) of Statement I, under circumstances such that eq (3) does not obtain. For if $L_{ab} = L_{ba}$, then $L_{ab}^* \equiv L_{ab} + W_{ab} \neq L_{ba}^*$ when $W_{ab} \neq 0$, and we need say nothing explicitly about the time-reversal symmetry of $X_a$ and $X_b$. Consequently, so long as any element of $(W_{ab})$ can be taken $\neq 0$, the question of time-reversal symmetry of the fluxes and forces is irrelevant to the arguments of Coleman and Truesdell (see footnote 5).

However, we should note that if no prior restriction is placed on the elements of $(W_{ab})$ and in particular we proceed to take them all $\neq 0$, then it is evident that if $J_a$ initially was definitely odd or even with respect to reversal of all the particle momenta, this will not be necessarily true of all the fluxes $J_a^*$. For if $J_a$ is odd and some of the $X_b$ are even, then $J_a^*$ will be a sum of odd and even terms, which itself has no definite time-reversal symmetry. Thus the transformation which leads to a counter-example for the broad statement of Meixner's theorem also produces fluxes which are neither symmetric or antisymmetric with respect to time inversion.

It is of interest, therefore, to examine the consequence of requiring that the fluxes and forces should all be even or odd, in order to determine whether this additional restriction is sufficient to assure that eqs (1) and (2) imply eqs (3) and (4). This restriction was indeed satisfied by the particular class of transformations considered by Meixner (see footnote 7), and it was taken by him to be a sufficient condition for the validity of "Meixner's theorem." Meixner's discussion [ref. 7, eq 8.3 and 8.4], however, assumes that the fluxes and forces are independent of one another, so that if $\dot{S} = \sum_a J_a X_a = \sum_a J_a' X_a$, then $J_a' = J_a$. Since $J_a$ and $J_a'$ are actually related to the $X_a$ by the phenomenological relations, Meixner's conclusion is not necessarily valid, and it is pertinent to reopen the question.

The particular physical motivation for this is that in a number of papers[8] designed to extend irreversible thermodynamics to include inertial effects involving very short times and very high sound frequencies, Statement I has been applied to phenomenological relations in which all quantities had a definite time-reversal symmetry [thus satisfying Meixner's restriction (see footnote 7)]. These phenomenological equations also have the property, which we shall see is crucial for the application of Onsager's theorem, that concentration and velocity gradients are not linked directly to any of the fluxes conjugate to such gradients. The latter restriction corresponds to the inertial principle that a fluctuation from chemical or thermal equilibrium does not instantaneously cause the conjugate reaction rate or heat flux, respectively, to differ from zero; and this may be expressed by setting $L_{ab} = 0$ when $X_a$ and $X_b$ are both even. For example, if $X_a$ and $X_b$ are proportional to concentration gradients in a multicomponent mixture, then the diffusion flux $J_a$ of component $a$ does not depend directly on $X_a$ or $X_b$, in accordance with the inertial principle that a concentration gradient does not produce an instantaneous flow.

In the present paper, we shall demonstrate that these additional restrictions on the phenomenological equations are indeed insufficient to assure the validity of Statement I. In fact, we must also require that all forces to which we seek to apply Onsager's theorem belong to one of two classes, of the type $\partial S/\partial y_a$ on the one hand, or concentration, velocity, or temperature gradients on the other, but no force can be a combination of the two types. This new restriction, together with limitations on the matrix $(L_{ab})$ imposed by the principles of inertia and irreversibility, will be formulated in a Statement II given at the end of the paper.

The proof of the new statement is fairly trivial, and the main body of the paper is devoted to showing that such additional restrictions on the types of forces are necessary. This is done by finding transformations from a set $J_a, X_a$ which satisfy eqs (1 to 4) to a set $J_a', X_a'$ which do not do so, even though all the forces are definitely even or odd, and $L_{ab} = 0$ when $X_a$ and $X_b$ are both even. It will be seen that the arguments of Coleman and Truesdell are not applicable in this case, and that the transformations to new forces and fluxes, $J_a'$ and $X_a'$, must be constructed in a more elaborate manner to be discussed in the following two sections.

[8] R. E. Nettleton, J. Chem. Phys. **33**, 237 (1960); Phys. Fluids **3**, 216 (1960); Phys. Fluids **4**, 74 (1961); Phys. Fluids **4**, 1488 (1961).

## 2. Formulation of Problem

In our construction of transformations which violate Meixner's theorem, we shall adopt conventional notation [ref. 1, p. 216] and denote even forces, $X_i(t)=X_i(-t)$, and their conjugate fluxes by Roman subscripts and odd forces, $X_\alpha(t)=-X_\alpha(-t)$, and their corresponding fluxes by Greek subscripts. The $\mathfrak{L}$-matrix, in this notation, can be expressed in terms of four submatrices in the form

$$\mathfrak{L}=\begin{bmatrix}(L_{ij}) & (L_{i\alpha})\\ (L_{\alpha i}) & (L_{\alpha\beta})\end{bmatrix} \tag{5}$$

where $L_{i\alpha}$ is the matrix element relating $J_i$ to $X_\alpha$ in the phenomenological eq (2), and the other submatrices are similarly defined. We shall assume that $\mathfrak{L}$ has the symmetry suggested by Onsager's theorem, i.e.,

$$L_{ij}=L_{ji}, \qquad L_{\alpha\beta}=L_{\beta\alpha}, \qquad L_{i\alpha}=-L_{\alpha i} \tag{6}$$

and we shall consider what happens to this symmetry under transformations to new fluxes $J_i^*$ and forces $X_i^*$ which are such that

$$\dot{S}=\sum J_iX_i+\sum_\alpha J_\alpha X_\alpha=\sum_i J_i^*X_i^*+\sum_\alpha J_\alpha^*X_\alpha^* \tag{7}$$

i.e., which keep $S$ invariant under the transformation.

Coleman and Truesdell (see footnote 5) have shown that a transformation of the type

$$\begin{aligned} X_i^*&\equiv X_i, & X_\alpha^*&\equiv X_\alpha\\ J_i^*&\equiv J_i+\sum_j W_{ij}X_j+\sum_\alpha W_{i\alpha}X_\alpha\\ J_\alpha^*&\equiv J_\alpha+\sum_j W_{\alpha j}X_j+\sum_\beta W_{\alpha\beta}X_\beta \end{aligned} \tag{8}$$

satisfies eq (7) provided the matrix $(W)$ is antisymmetric, i.e., $W_{ij}=-W_{ji}$, $W_{\alpha\beta}=-W_{\beta\alpha}$, and $W_{i\alpha}=-W_{\alpha i}$. It is also clear that (8) will destroy the symmetry given by eq (6), since the transformed $\mathfrak{L}$−matrix, $\mathfrak{L}^*$, has the property

$$L_{ij}^*=L_{ij}+W_{ij}\neq L_{ji}^*. \tag{9}$$

Let us remark here, however, that ordinarily $J_i$ has a definite time-reversal symmetry opposite to that of its conjugate force $X_i$. For example, this is the case when the flux is a rate of change, e.g., $\dot{y}_\alpha$, of one of the variables on which the second depends linearly—the case usually discussed in proofs of Onsager's theorem [compare ref. 1, pp. 6, 7]. But although we may have $J_i(t)=-J_i(-t)$, we can still assign no definite time-reversal symmetry to $J_i^*$ given by eq (8) unless $W_{ij}=0$ for all pairs of Roman indices $i,j$. Similar considerations show that if $J_\alpha(t)=J_\alpha(-t)$, then $J_\alpha^*$ will be neither symmetric nor antisymmetric unless $W_{\alpha\beta}=0$ for all Greek indices $\alpha,\beta$. But if $W_{ij}=0=W_{\alpha\beta}$ for all pairs $(i,j)$ and $(\alpha,\beta)$, then $L_{ij}^*=L_{ij}=L_{ji}^*$, $L_{\alpha\beta}^*=L_{\alpha\beta}=L_{\beta\alpha}^*$, and $L_{i\alpha}^*=L_{i\alpha}+W_{i\alpha}=-L_{\alpha i}^*$, so that the symmetry given by eq (6) is retained by the transformed $\mathfrak{L}$-matrix. Consequently, the transformation proposed by Coleman and Truesdell will not destroy the Onsager symmetry of the $\mathfrak{L}$-matrix if it does not destroy the time-reversal symmetry of the fluxes $J_i$.

It is for this reason that, as we remarked in the previous section, the Coleman-Truesdell demonstration is of no use in judging the applicability of Statement I to cases where the fluxes as well as the forces are all either odd or even with respect to time-reversal. The type of transformation which does prove useful for this purpose is one in which the fluxes $J_i^*$ are sums of fluxes $J_i$ and the $J_\alpha^*$ depend only on the $J_\alpha$, and similarly for the forces. If this can be done in such a way that eqs (1) and (2) hold for the transformed fluxes and forces when all quantities in these equations are starred, and yet eq (3) does not hold, then we shall have shown that Statement I must be modified before Onsager's theorem can be applied to the thermodynamic rate equations which have been derived (see footnote 8) to describe inertial effects in fluids. There the fluxes and forces all have a definite time-reversal symmetry, and temperature and concentration gradients are coupled to forces of the type $\partial S/\partial y_\alpha$, so that existing proofs (see footnotes 1, 6) of Onsager's theorem do not hold automatically.

Our desire to discuss inertial effects is also the reason we cannot invert the phenomenological equations and solve for the forces $X_i$ in terms of the fluxes $J_i$. If this could be done, the result could be substituted into eq (8) with $W_{i\alpha}=0$, and the Coleman-Truesdell transformation would reduce to an expression relating $J_i^*$ to a linear sum of fluxes $J_i$. There would then be nothing further to demonstrate by considering the transformation introduced in the following section [eq (10)].

However, if we are going to describe extremely short time intervals or high acoustic frequencies, we cannot have a phenomenological equation of the type which can be inverted, having the form

$$J_i=\sum_j L_{ij}X_j$$

in which $L_{i\alpha}=0=L_{\alpha i}$, and the $X_\alpha$ do not appear. For if the motion is suddenly reversed, the right-hand member of this equation would be unchanged, while the flux $J_i$ would reverse its sign. This could be true only if $J_i=0$ at very high frequencies. Since it is the high-frequency components of the Fourier expansion of the motion which determine the behavior of the system over very short time intervals, the conclusion $J_i=0$ at high frequencies implies that the phenomenological equations give no short-time response to the imposition of thermodynamic forces, and thus they do not have an essential property of equations which include inertial effects. Consequently, we never have all the matrix elements $L_{\alpha i}=0$ when we describe the very short-time behavior of a physical system. This, in turn, means

that it is impossible in general to solve for $X_i$ in terms of the fluxes $\{J_k\}$ alone or for $X_\alpha$ in terms of the fluxes $\{J_\beta\}$. Inversion of the phenomenological equations describing inertial effects expresses each force as a sum of fluxes some of which will always have the same time-reversal symmetry as the force in question. Substitution of the result into eqs (8) with $W_{ij}\neq 0$ leads to a $J_i^*$ which depends linearly on both $J_i$ and $J_\alpha$. Thus for at least one class of phenomenological equations which are of physical interest, i.e., those which include inertial effects, there is an essential difference between the transformations discussed in the present paper and those of Coleman and Truesdell.

## 3. New Proof of Coleman-Truesdell Theorem

In the present section we shall discuss a very wide class of transformations which have the property that they conserve time-reversal symmetry of all fluxes and forces. It will be shown that transformations with this property may be so constructed that, while they preserve the antisymmetric Onsager relation, so that $L_{i\alpha}^*=-L_{\alpha i}^*$, nevertheless they lead to $L_{\alpha\beta}^*\neq L_{\beta\alpha}^*$. In accordance with the discussion of the preceding section, we shall assume $L_{i\alpha}\neq 0$ for some $i$ and $\alpha$, so that we have in mind the phenomenological description of inertial effects, and so that the transformations here proposed which conserve time-reversal symmetry cannot be reduced to the form assumed by Coleman and Truesdell. Furthermore, we shall take $L_{ij}=0$ for all Roman indices $i,j$, since, as was pointed out in the introduction, this is always a property of the phenomenological equations which describe inertial effects. This gives the appearance of restricting the proof to a very special class of phenomena. However, the description of inertial effects is the only existing instance in the scientific literature in which a solid physical basis has been established for the existence of antisymmetric reciprocal relations, and so the assumption $L_{ij}=0$ for all $i$ and $j$ appears physically to be quite general under circumstances such that some $L_{i\alpha}\neq 0$.

The particular transformations which we shall discuss are the following:

$$X_i^*=\sum_j A_{ij}X_j, \qquad X_\alpha^*=\sum_\beta A_{\alpha\beta}X_\beta$$

$$J_i^*=\sum_j B_{ij}J_j \qquad J_\alpha^*=\sum_\beta B_{\alpha\beta}J_\beta. \tag{10}$$

This assures that $X_i^*$, $J_\alpha^*$ are even functions of time and $X_\alpha^*$, $J_i^*$ odd functions, provided this was true of the corresponding unstarred quantities. We define the matrices $\mathfrak{A}$, $\mathfrak{B}$, and $\mathfrak{C}$, by the equations

$$\mathfrak{A}\equiv\begin{bmatrix}(A_{ij}) & 0\\ 0 & (A_{\alpha\beta})\end{bmatrix} \qquad \mathfrak{B}\equiv\begin{bmatrix}(B_{ij}) & 0\\ 0 & (B_{\alpha\beta})\end{bmatrix} \tag{11}$$

$$\mathfrak{A}\equiv\mathfrak{B}'^{-1}(\mathfrak{J}+\mathfrak{C}) \tag{12}$$

where the prime denotes the transposed matrix, and $\mathfrak{J}$ is the identity. This definition assumes that the matrix $\mathfrak{B}$ is nonsingular, which is always possible since $\mathfrak{B}$ is arbitrary.

On substituting eqs (10) into eq (7) and using the phenomenological relations between fluxes and forces to express $\dot{S}$ as a quadratic form depending only on the forces $X_i$ and $X_\alpha$, we readily find that eq (7) can hold if, and only if,

$$\mathfrak{C}'\mathfrak{L}+\mathfrak{L}'\mathfrak{C}=0. \tag{13}$$

At the same time, eqs (10) imply that the transformed $\mathfrak{L}$-matrix is

$$\mathfrak{L}^*=\mathfrak{B}\mathfrak{L}\mathfrak{A}^{-1}=\mathfrak{B}\mathfrak{L}(\mathfrak{J}+\mathfrak{C})^{-1}\mathfrak{B}' \tag{14}$$

provided $\mathfrak{C}$ is chosen so that $\mathfrak{A}$ is nonsingular. From eq (14) we readily see that the symmetry properties of $\mathfrak{L}^*$ are identical with those of $\mathfrak{L}(\mathfrak{J}+\mathfrak{C})^{-1}$, which we shall now examine.

$\mathfrak{L}$ is uniquely decomposable into the sum of a symmetric matrix $\mathfrak{L}_s$ and an antisymmetric matrix $\mathfrak{L}_a$, where

$$\mathfrak{L}_s=\begin{bmatrix}(L_{ij}) & 0\\ 0 & (L_{\alpha\beta})\end{bmatrix} \qquad \mathfrak{L}_a=\begin{bmatrix}0 & (L_{i\alpha})\\ (L_{\alpha i}) & 0\end{bmatrix}. \tag{15}$$

One can readily show that multiplication of $\mathfrak{L}_s$ by $\mathfrak{C}$ or $\mathfrak{C}'$ results in a matrix of the same type, with zeros in all the entries designated by zeros in eq (15). Similarly for $\mathfrak{L}_a$. Substitution of eq (15) into eq (13) then gives

$$\mathfrak{C}'\mathfrak{L}_a-\mathfrak{L}_a\mathfrak{C}=0 \tag{16}$$

$$\mathfrak{C}'\mathfrak{L}_s+\mathfrak{L}_s\mathfrak{C}=0. \tag{17}$$

Equation (16) further implies

$$\mathfrak{L}_a(\mathfrak{J}+\mathfrak{C})^{-1}=-(\mathfrak{J}+\mathfrak{C}')^{-1}\mathfrak{L}_a'. \tag{18}$$

From eq (14) we have:

$$\mathfrak{B}\mathfrak{L}_a(\mathfrak{J}+\mathfrak{C})^{-1}\mathfrak{B}'=\begin{bmatrix}0 & (L_{i\alpha}^*)\\ (L_{\alpha i}^*) & 0\end{bmatrix}. \tag{19}$$

Equations (18) and (19) show that the transformation (10) preserves the antisymmetric Onsager relation $L_{i\alpha}^*=-L_{\alpha i}^*$. We can, however, find a solution of eqs (16) and (17) such that $L_{\alpha\beta}^*\neq L_{\beta\alpha}^*$, i.e., the Onsager symmetry is destroyed by the transformation. To this end, let us proceed to introduce the assumption that

$$L_{ij}=0 \text{ (all } i,j). \tag{20}$$

This, as was previously pointed out, is the form of $\mathfrak{L}$-matrix used in earlier papers on inertial effects (see footnote 8). When eq (20) is substituted into eq (17), we find the solution ($C_{ij}$ and $C_{\alpha\beta}$ are elements of $\mathfrak{C}$):

$$C_{\alpha\beta}=\sum_\gamma W_{\alpha\gamma}L_{\gamma\beta} \tag{21}$$

where $(W_{\alpha\beta})$ is any antisymmetric matrix. From

eq (16), we obtain

$$\sum_j C_{ji}L_{j\beta}=\sum_\gamma L_{i\gamma}C_{\gamma\beta}. \tag{22}$$

Equation (22) will, in general, possess a solution provided the number of indices $\beta$ does not exceed the number of Roman indices.

The existence of solutions of eqs (21) and (22), however, is, in general, incompatible with the symmetry of the matrix

$$\mathfrak{B}\mathfrak{L}_s(\mathfrak{Z}+\mathfrak{C})^{-1}\mathfrak{B}'=\begin{bmatrix}(L^*_{ij}) & 0\\ 0 & (L^*_{\alpha\beta})\end{bmatrix}.$$

If $\mathfrak{L}_s(\mathfrak{Z}+\mathfrak{C})^{-1}$ equals its transpose, $(\mathfrak{Z}+\mathfrak{C}')^{-1}\mathfrak{L}_s$, then eq (17) implies $\mathfrak{L}_s\mathfrak{C}=0$, which is not satisfied by eq (21) for nonzero $(W_{\alpha\beta})$.

A simple choice for $W_{\alpha\beta}$ may be made in the event, which frequently occurs in practice, that $(L_{ij})$ and $(L_{\alpha\beta})$ are both square matrices of the same even number $n\geq 2$ of rows [compare reference 8], so that $(C_{ij})$ and $(C_{\alpha\beta})$ are also $n$-rowed square matrices. In this case we can pick

$$W_{\alpha\beta}=\epsilon_1(1-\delta_{\alpha\beta})\ \mathrm{sgn}\ (\beta-\alpha)$$

where $\delta_{\alpha\beta}$ is a Kronecker delta. Then from eqs (21) and (22),

$$\det\mathfrak{C}=[\det(C_{\alpha\beta})]^2=[\epsilon_1^n\det[(L_{\alpha\beta})]]^2$$

provided the matrix $(L_{i\beta})$ is nonsingular. When $(L_{ij})=0$, the latter condition must hold in order that the fluxes $J_i(i=1,\ .\ .\ .,\ n)$ be linearly independent. There must also exist cases in which, at the same time, $\det(L_{\alpha\beta})\neq 0$, since this determinant (see footnote 8) is the product of the relaxation frequencies for the inertial effects. Thus there will exist solutions of eqs (21) and (22) for which $\det\mathfrak{C}$ has an arbitrary positive value. The same is easily shown to hold for $\det(\mathfrak{Z}+\mathfrak{C})$.

The latter result in turn assures that we can find a transformation such that the $J_i^*,J_\alpha^*$ are linearly independent, and similarly the $X_i^*,X_\alpha^*$. For the matrix $\mathfrak{B}$ is arbitrary, and thus can be chosen to be nonsingular, and then from eq (12)

$$\det\mathfrak{A}=(\det\mathfrak{B})^{-1}\det(\mathfrak{Z}+\mathfrak{C})$$

which can be made $\neq 0$ by proper choice of $\epsilon_1$.

Thus we have established that starting with a linearly independent set of forces and fluxes, $J_i$, $J_\alpha$, and $X_i,X_\alpha$, with an $\mathfrak{L}$-matrix which satisfies eq (16) and has the properties found to hold (see footnote 8) in a phenomenological theory that includes inertial effects, i.e., $L_{ij}=0$ and $L_{i\alpha}\neq 0$ for some $i$ and $\alpha$, we can transform to a new linearly-independent set, $J_i^*,J_\alpha^*$ and $X_i^*,X_\alpha^*$, having definite time-reversal symmetry and such that $L^*_{\alpha\beta}\neq L^*_{\beta\alpha}$. This shows that Meixner's theorem does not hold in general even under these circumstances, although the Coleman-Truesdell proof does not apply here, and so we must seek a modification of Statement I.

## 4. Reformulation of Meixner's Theorem

It is evident from existing proofs of Onsager's theorem [ref. 1, chap. 2 and 11] that if there exists a set of symmetric state variables $y_i(i=1,\ .\ .\ .,\ n_1)$, and antisymmetric variables $y_\alpha(\alpha=1,\ .\ .\ .,\ n_2)$ such that $X_i=\partial S/\partial y_i$, $J_i=\dot{y}_i$ (all $i$), while $X_\alpha=\partial S/\partial y_\alpha$, $J_\alpha=\dot{y}_\alpha$ (all $\alpha$), and there are no nonzero forces except these, then Statement I is true. Furthermore, Statement I remains true if we transform from one set of fluxes and forces of this type to another set of the same type, and so Meixner's theorem is also true under this additional restriction.

The situations where Meixner's theorem has been found to fail arise where some or all of the forces $X_\alpha$ are not entropy derivatives of the form $\partial S/\partial y_\alpha$. For example, it follows from eq (12) that whenever the $\mathfrak{L}$-matrix has the right properties so that $\mathfrak{C}$ does not vanish identically, then we can have a transformation [including an additional set of equations not shown in which the Greek subscripts $\alpha,\beta$ are replaced by Roman subscripts $i,j$]:

$$J_\alpha^*\equiv\dot{y}_\alpha^*=\sum_\beta B_{\alpha\beta}\dot{y}_\beta,\ \dot{y}_\beta\equiv J_\beta$$

and

$$X_\alpha^*=\partial S/\partial y_\alpha^*+\sum_{\beta,\gamma}(B)^{-1}_{\beta\alpha}C_{\beta\gamma}X_\gamma$$

where

$$y_\alpha^*=\sum_\beta B_{\alpha\beta}y_\beta,\ \text{and}\ \partial S/\partial y_\alpha^*=\sum_\beta(\partial S/\partial y_\beta)(\partial y_\beta/\partial y_\alpha^*)$$
$$=\sum_\beta(B^{-1})_{\beta\alpha}X_\beta.$$

The transformed forces and fluxes of this type indeed satisfy eqs (7) and (10), and yet we have seen that $L^*_{\alpha\beta}\neq L^*_{\beta\alpha}$.

At this point we must not lose sight of the fact that in order to obtain a nonzero matrix $(C_{\alpha\beta})$, and thus to find a counter-example to Meixner's theorem under the restriction to transformations of the form of eq (10), we had to assume $(L_{ij})=0$, which has thus far been shown to hold in general only for the phenomenological equations which describe inertial effects (see footnote 8). In the latter case, when the system is a small subvolume immersed in an infinite fluid [compare ref. 8], we have to deal with two types of forces. First of all, there are the forces of the type $\partial S/\partial y_i$ and $\partial S/\partial y_\alpha$ which are functions of the state variables $y_i$ and $y_\alpha$ on which $S$ depends explicitly. Secondly, there are forces such as temperature, velocity, and concentration gradients on which $S$ does not depend explicitly and which are coupled in the phenomenological equations to forces which are of the type $\partial S/\partial y_i$ or $\partial S/\partial y_\alpha$. This is a state of affairs which arises because over very short times the system evolves independently of its environment, and it is only after one or more molecular collision times that the effect of gradients of temperature or velocity begins to be felt. Thus the entropy does not depend explicitly on these gradients, but instead on the variables $y_i$ and $y_\alpha$ which are related to the gradients through the phenomenological equations [compare ref. 8].

Because of the existence of these two types of forces, we can construct forces $X_\alpha^*$ which are not of the type $\partial S/\partial y_\alpha^*$ by adding gradients of velocity to the forces $\partial S/\partial y_\alpha$ as well as by combining the latter among themselves. Thus, in general, we cannot expect that $L_{\alpha\beta}=L_{\beta\alpha}$ if either $X_\alpha$ or $X_\beta$ involves any velocity, concentration, or temperature gradients which do not appear explicitly in $S$, although $L_{\alpha\beta}$ is equal to $L_{\beta\alpha}$ whenever $J_\alpha=\dot{y}_\alpha$, $J_\beta=\dot{y}_\beta$, $X_\alpha=\partial S/\partial y_\alpha$, and $X_\beta=\partial S/\partial y_\beta$. In other words, it is possible, by combining forces of the first type (gradients) with forces of the second type ($\partial S/\partial y_i$, etc.) to construct transformations which violate Meixner's theorem.

In addition to the demonstration that

(a) transformations can be found which keep $\dot{S}$ invariant, but which lead to an $\mathfrak{L}$-matrix in which $L_{\alpha\beta}\neq L_{\beta\alpha}$, if, and only if, not all transformed fluxes are time-derivatives $\dot{y}_i$, $\dot{y}_\alpha$ or not all transformed fluxes are entropy derivatives $\partial S/\partial y_i$, $\partial S/\partial y_\alpha$, we have shown that

(b) if eqs (6) hold for a particular choice of $J_i$, $X_i$, $J_\alpha$, $X_\alpha$, then the antisymmetric relation $L_{i\alpha}=-L_{\alpha i}$ is conserved by any transformation of the form of eq (10) which keeps $\dot{S}$ invariant, without further restriction on the $J_i$,$X_i$ and $J_\alpha$,$X_\alpha$.

These two observations provide a new statement of Meixner's theorem.

Since the broad (and incorrect) statement of Meixner's theorem referred to in the introduction has often been advanced as a foundation for statement I of Onsager's theorem, we might jump to the conclusion that statements (a) and (b) of the reformulated Meixner's theorem are sufficient to provide the basis for a revision of Statement I. However, statement (b) does not provide for transformations more general in form than eq (10). Specifically, it omits transformations in which $J_i^*$ is a linear combination of forces $X_\alpha$ in addition to fluxes $J_i$. Such transformations preserve the time-reversal symmetry of the transformed fluxes, although they are more general than eq (10). In order to cover all possible cases, it appears to be necessary to require:

(1) $\mathfrak{L}$ must be the matrix of a positive definite quadratic form.

(2) Only those transformations are considered for which $L_{ij}^*=0$ (all $i,j$).

These physical restrictions are related respectively to the definition of irreversibility and to the inertial properties of mechanical systems [compare ref. 8]. When they obtain, they imply trivially that $L_{i\alpha}^*=-L_{\alpha i}^*$, since otherwise $\dot{S}$ would contain cross terms of the type $X_i^*X_\alpha^*$ and no term in $(X_i^{*2})$, so that for $X_i^*$ sufficiently large, reversal of the signs of the forces $X_\alpha^*$ could make $\dot{S}<0$.

Actually, as we have already remarked at the beginning of the third section, condition (2) that $L_{ij}^*=0$ is probably superfluous, since this condition is always satisfied by phenomenological equations describing inertial effects, and, in the latter case, no transformation can be found which violates it. Furthermore, no physical examples except the inertial rate equations have ever been demonstrated for which $L_{i\alpha}\neq 0$ for some $i$ and $\alpha$. Nevertheless, we shall provide for the possible existence of cases where $L_{i\alpha}\neq 0\neq L_{ij}$ in Statement II. There we shall require specifically that $(L_{ij})=0$ whenever we extend Onsager's theorem to the coefficients which couple gradient-type forces to forces of the type $\partial S/\partial y_\alpha$.

On the basis of these observations, we can restate Onsager's theorem in the following form, which covers the case in which there are fluxes which are not time-derivatives of state variables, or forces which are not derivatives of $S$ with respect to associated state variables:

Statement II: Suppose the entropy $S$ in a thermodynamic system is a function of a set of $N$ state variables,[9] of which $n_1$ variables $y_i(i=1,\ldots,n_1)$ are symmetric with respect to time-reversal and $n_2=N-n_1$ state variables $y_\alpha(\alpha=1,\ldots,n_2)$ are antisymmetric. With these variables we associate the forces $X_i$,$X_\alpha$ and fluxes $J_i$,$J_\alpha$ given by [10]

$$J_i\equiv\dot{y}_i,\ X_i\equiv\partial S/\partial y_i(i=1,\ldots,n_1)$$

$$J_\alpha\equiv\dot{y}_\alpha,\ X_\alpha\equiv\partial S/\partial y_\alpha(\alpha=1,\ldots,n_2).$$

Suppose also that $\dot{S}$, which denotes the irreversible part of the time-rate of change of $S$, is given by

$$\dot{S}=\sum_{i=1}^{m_1}J_iX_i+\sum_{\alpha=1}^{m_2}J_\alpha X_\alpha$$

where $m_1\geq n_1$ and $m_2\geq n_2$, and the $m_1$ quantities $J_i$ and $m_2$ quantities $J_\alpha$ are all linearly independent, and similarly for the $X_i$, $X_\alpha$. This provides for the case where there may be forces such as temperature, velocity, or concentration gradients on which $S$ does not depend explicitly, these being denoted by subscripts $n_1<i\leq m_1$, $n_2<\alpha\leq m_2$. It is assumed that the $X_i(1\leq i\leq m_1)$ are all symmetric with respect to time-reversal and the $X_\alpha(1\leq\alpha\leq m_2)$ are all antisymmetric. Then if

$$J_i=\sum_{j=1}^{m_1}L_{ij}X_j+\sum_{\alpha=1}^{m_2}L_{i\alpha}X_\alpha$$

$$J_\alpha=\sum_{j=1}^{m_1}L_{\alpha j}X_j+\sum_{\beta=1}^{m_2}L_{\alpha\beta}X_\beta$$

we may assert, in the absence of a magnetic field, that

$$L_{ij}=L_{ji}(i\leq n_1,\ j\leq n_1)$$

$$L_{\alpha\beta}=L_{\beta\alpha}(\alpha\leq n_2,\ \beta\leq n_2)$$

$$L_{i\alpha}=-L_{\alpha i}(i\leq n_1,\ \alpha\leq n_2$$

and for all $i,\alpha$ whenever $L_{ij}=0$ for all $i,j$).

[9] It must be remembered here that these $N$ variables are not necessarily the entire set of state variables appearing in the phenomenological equations, since we have pointed out that there may be others on which $S$ does not depend explicitly. These additional variables, which are usually gradients, characterize the state of the surroundings of the system, and they affect the rates of change of the variables $y_i$,$y_\alpha$, even though the entropy and free energy depend only on the latter.

[10] In an open system $\dot{y}_i$ and $\dot{y}_\alpha$ denote only those parts of the time-rates of change which are due to irreversible processess within the system, and exclude the contribution to changes in fluid composition or structure attributable to molecular diffusion across the boundary.

## 5. Discussion

The last result in Statement II expresses the fact, which has already been mentioned, that when $L_{ij}=0$ (all $i,j$), we must have $L_{i\alpha}=-L_{\alpha i}$ in order to assure that $\dot{S}$ contains no cross terms $X_iX_\alpha$; for otherwise $\dot{S}$ would not be a positive definite quadratic form in the forces. The other two symmetric reciprocal relations, $L_{ij}=L_{ji}$ and $L_{\alpha\beta}=L_{\beta\alpha}$, involve only the forces associated with the $N$ state variables, and not the remaining $m_1+m_2-N$ forces, since there exists no proof of symmetric Onsager relations for the latter. Indeed we can find transformations involving the gradient-type forces which are such that Onsager symmetry as expressed by Statement I is violated. It has also been shown that transformations contradicting Statement I can be found even when the gradients all vanish, and so we have been careful in formulating Statement II to restrict those forces $X_\alpha$ and $X_\beta$ for which we assert $L_{\alpha\beta}=L_{\beta\alpha}$ to the ones which can be written in the form $\partial S/\partial y_\alpha$ and $\partial S/\partial y_\beta$. However, in the existing applications (see footnote 8) of Onsager's theorem to rate equations describing inertial effects, those forces which did not depend on gradients not appearing in $S$ were always so constructed as to be of the form $\partial S/\partial y_i$, $\partial S/\partial y_\alpha$. Therefore, it is only with respect to the presence of velocity, concentration (in binary mixtures), and temperature gradients that serious questions have arisen over past usage of Statement I.

If a concentration or temperature gradient should appear in the phenomenological equations multiplied by a coefficient of the type $L_{ij}$, then statement II would tell us nothing about the symmetry of this coefficient with respect to interchange of its indices. In previous applications (see footnote 8) it has always been possible to write phenomenological equations in such a form that this situation did not arise, and so that $L_{ij}=0$ for all $i,j$, thus providing no circumstances not covered by statement II. It appears that, if we take account of inertial effects in such processes as heat conduction and diffusion, then we can always avoid the presence of terms concerning which statement II says nothing, although no completely general demonstration of this apparent fact exists.

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# Selected Bibliography of Statistical Literature 1930 to 1957: VI. Theory of Estimation and Testing of Hypotheses, Sampling Distributions, and Theory of Sample Surveys

This is the sixth in a series of bibliographies that deal with various specific subjects in the field of statistics. Given here are approximately sixteen hundred references and titles to important publications dealing with the theory of statistical estimation and testing of hypotheses (parametric case), sampling distributions, and the theory of sample surveys.

Lola S. Deming

(May 22, 1962)

The purpose of any bibliography is to provide a search of the literature for students in a particular field of investigation. This series provides such a service in several of the specific subjects within the very large field of probability and mathematical statistics. Search of the literature in this case has been of a parasitic nature, however, in that two prominent reviewing journals have been used for our source material. Abstracts of original papers appearing in a myriad of technical journals throughout the world and in many languages, have been transferred from the reviewing journals to cards which form a collection of statistical material maintained on a current basis in the NBS Statistical Engineering Laboratory. *Zentralblatt für Mathematik* was used for this purpose from 1930 to 1939; *Mathematical Reviews* has been used from 1940 onward. The abstracts have been coded into categories of subject matter following generally, but with some minor deviations, the classification scheme of *Mathematical Reviews*. One abstract may be classified under several subjects; hence may appear in more than one place in this series of bibliographies. The references given in these published bibliographies were transcribed from the abstracts by means of punched cards thereby necessitating severe and unconventional abbreviations in many cases. The titles were added as each bibliography was compiled.

The present bibliography is a composite of several areas in statistics because of the unusual amount of straddling here. In addition to the papers listed by the reviewing journals directly under these subjects, we have also combed through those classified by *Mathematical Reviews* under the more general headings *General Statistics* and *Statistical Tests and Related Topics*. Papers dealing solely with ranking and paired-comparison techniques, order statistics, and other distribution-free methods have been excluded for the most part, in view of the thorough and extensive coverage of these areas by Savage's [1] latest bibliography of nonparametric statistics.

The abstracting services of both *Zentralblatt für Mathematik* and *Mathematical Reviews* have favored papers and books on theory and methodology rather than applications. Therefore, we do not claim completeness, especially in the more practical aspects of statistics. This becomes particularly evident in the field of sample surveys. Furthermore, there does not seem always to be a clear distinction between writings dealing with sampling in the sense of the theory and methodology of sample surveys and those dealing with sampling distributions of particular functions of sample values in successive random samples of fixed size from a specified probability distribution.

Because of the overlapping nature of the material included in this bibliography, we have made no effort at subclassifying the references.

The organization of information within each entry is:

*Author:* The author's surname, followed by initials only. In the case of multiple authorships, the journal reference appears with each author's name, but the title of the paper appears with the first author only. The symbol ♦ preceding the surname denotes multiple authorship.

*Title:* Exactly as in the reviewing journal. Titles of separately bound publications (books, reports, theses, etc.) are in italics, followed by the publisher.

*Reference to literature:* The name of the journal in italics, the number of the volume in bold face, the initial page number, and the date of publication in parentheses comprise the reference to the original article.

*Reference to the abstract:* The final symbols M (for *Mathematical Reviews*) and Z (for *Zentralblatt für Mathematik*) are followed by the volume number and page number of the reviewing journal in which the abstract appears.

[1] I. Richard Savage, *Bibliography of Nonparametric Statistics* published by the Harvard University Press, Cambridge, Mass., 1962. This is a much extended version of his earlier *Bibliography of Nonparametric Statistics and Related Topics*, J. Amer. Statist. Assoc. Vol. 48, pp. 844-906, December 1953.

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642319—62——5

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(Paper 66B3–80)

JOURNAL OF RESEARCH of the National Bureau of Standards—B. Mathematics and Mathematical Physics
Vol. 66B, No. 3, July–September 1962

# Publications of the National Bureau of Standards *

## Selected Abstracts

**Derivation of the relaxation spectrum representation of the mechanical response function,** R. S. Marvin, *J. Research NBS* **66A** (*Phys. and Chem.*) *No. 4 (July–Aug. 1962).*
Relaxation spectra have been used in both the presentation and interpretation of measurements of the mechanical properties of rubberlike polymers.

**Analysis of coaxial two-terminal conical capacitor,** M. C. Selby, *NBS Mono. 46, April 6, 1962, 20 cents.*
Adjustable capacitors having electrodes in the form of coaxial cones or frustums have been used on rare occasions in the past; but their potential superiority to other types of capacitors for some important applications have been overlooked. The advantage of this geometry over cylindrical or disk forms is that the practical capacitance range is several times larger. An example cites the capacitance ranges of a disk, cylindrical, and conical type to be 10, 40, and 168 to one, respectively. An approximate equation was derived for this conical capacitor and close agreement is shown between computed and measured values of capacitance versus electrode displacement. Multiple cone and different shape electrodes are suggested to obtain large values of capacitance with an appreciable saving of space and further increased range of capacitance. The electric field is plotted and its construction steps for axial symmetry are given.

**Evaluation of convolution integrals occurring in the theory of mixed path propagation,** J. R. Johler and C. M. Lilley, *NBS Tech. Note 132 (PB161633) (1961) $1.00.*
The theory of propagation of electromagnetic waves around a sphere treats the smooth homogeneous case, i.e., the case in which the surface impedance of the sphere is uninterrupted by an abrupt change in conductivity such as a land sea boundary. It is known, however, that such a theory can be extended to treat inhomogeneous, irregular terrain by formulating certain convolution integrals which utilize the smooth homogeneous formulas. The evaluation of these integrals can be accomplished with dispatch on a large-scale electronic computer with the aid of numerical analysis techniques.
The particular case of a land/sea boundary in a smooth, spherical surface is illustrated for a variety of cases by evaluating the convolution integrals on a large-scale computer.

**Displacement and strain-energy distribution in a longitudinally vibrating cylindrical rod with a viscoelastic coating,** P. Hertelendy, *J. Appl. Mechanics, No. 61–WA–30 (1962).*
A numerical solution by R. M. Davies of the Pochhammer frequency equation is used to determine the displacement and strain-energy distribution across the cross section of an infinite elastic circular cylindrical rod for a number of wave lengths of the first, second and third modes of symmetrical longitudinal wave propagation. With these results the effect of a thin uniform layer of viscoelastic material is investigated. The four viscoelastic parameters of the coating are reduced to one in the definition and computation of upper and lower bounds of the loss factor, and the application of results to experimental work is discussed.

**A string language for symbol manipulation based on ALGOL 60,** J. H. Wegstein and W. W. Youden, *Commun. ACM* **5**, *No. 1, 54–61 (Jan. 1962).*
An artificial computer programming language is proposed for describing the manipulation of strings of characters and symbols. The concept of strings, introduced in the ALGOL 60 Report, is extended by adding: (1) the declaration of strings, substrings, and string arrays with explicit lengths; (2) the ability to concatenate and shift strings; and (3) the ranking of symbols for comparing strings in Boolean relations. A primer or informal description of the language is followed by examples, a description of experiments with the language on an IBM 704 computer, and a formal description which, taken with the ALGOL 60 Report, defines the proposed string language.

**Statistical problems arising in the establishment of physical standards,** W. J. Youden, *Proc. Fourth Berkeley Symp. on Math. Statistics and Probability III, 321–335 (1961).*
The establishment and maintenance of physical standards is indispensable for scientific research, commerce, and industry. The first standards were sufficiently ahead of the existing needs so that questions of precision and accuracy were hardly raised. In recent years the requirements of research and industry have become extremely exacting. Questions of precision and accuracy are now raised on every hand.
Among the statistical problems connected with the development of improved physical standards are the estimation of measurement precision; the design of experiments to provide information on the accuracy; and the reconciliation of results obtained in the national laboratories of the countries undertaking this work. This paper reviews and illustrates by actual examples, some of the ways in which statistical methodology can make contributions in this field.

**A lattice with an unusual frequency spectrum,** R. J. Rubin and R. Zwanzig, *J. Math. Phys.* **2**, *No. 6, 861–864 (Nov.-Dec. 1961).*
The lattice is a special rooted Cayley tree, generated by $N$ successive $m$-fold branchings. With each point of the tree are associated a mass $M$ and a position coordinate $x_i$. All end points are held fixed at $x_i=0$. The potential energy is $V=\frac{1}{2}\sum_{ij}K_{ij}(x_i-x_j)^2$, where $K_{ij}=K$ if $i$ and $j$ are connected neighbors and neither is an end point, $K_{ij}=\alpha K$ if $i$ and $j$ are connected neighbors and either is a branch tip point, and $K_{ij}=0$ if $i$ and $j$ are not connected neighbors. The allowed frequencies of vibration are obtained for two different cases; In the first case all springs are identical ($\alpha=1$), and in the second case the springs connecting interior points to the branch tips are cut ($\alpha=0$). In the case in which all force constants are the same, the allowed frequencies of vibration, in the limit of infinite, $N$, are given by $\omega(r)=(K/M)^{\frac{1}{2}}[m+1-2m\cos r\pi]^{\frac{1}{2}}$, where $r$ is any rational number between zero and one. The fraction of all normal modes having precisely the value $\omega(r)$ is $\rho[\omega(r)]=(m^2-1)^2/(m^q-1)$, where $r$ is expressed as the ratio $r=p/q$ of relatively prime integers $p$ and $q$. The frequency spectrum is dense within the interval $(m-1,\ m+1)$; and $\rho[\omega]$ is discontinuous at every $\omega$ for which it does not vanish.

**A new approach to the mechanical syntactic analysis of Russian,** I. I. Rhodes, *Mech. Transl.* **6**, *33–50 (Nov. 1961).*
This paper categorically rejects the possibility of considering a word-to-word conversion as a translation. A true translation is unattainable, even by the human agent, let alone by mechanical means. However, a crude *practical* translation is probably achievable. The present paper deals with a scheme for the syntactic integration of Russian sentences.

**Congruences for the partition function to composite moduli,** M. Newman, *Illinois J. Math.* **6**, *No. 1, 59–63 (Mar. 1962).*
The principal result proved is that the unrestricted partition function $p(n)$ fills all residue classes modulo 65 infinitely often. Similar results are proved.

**The shape of the geomagnetic field boundary under uniform external pressure,** R. J. Slutz, *J. Geophys. Research* **67,** *No. 2, 505–513 (Feb. 1962).*
A solution is given for the shape of the cavity which separates the earth's magnetic field from the interplanetary plasma, for a model which assumes the plasma pressure to be constant over the surface of the cavity (thus giving axial symmetry). It is seen that along the polar axes there are cusps extending inward which reduce the cavity size along these axes to $\frac{2}{3}$ of the size in the equatorial plane. Applying these results as an approximation in the earth-sun direction for the non-axially-symmetric case of solar wind gives an estimate of about 9 earth radii for the distance of the boundary from the earth's center in the direction of the sun.

**Inequalities for the permanent function,** M. Marcus and M. Newman, *Ann. Math.* **75,** *No. 1, 47–62 (Jan. 1962).*
By exhibiting the permanent function as an inner product on a suitably defined space of tensors on a unitary space, many inequalities and bounds for the permanent are obtained. Thus it is shown that if the $n \times n$ matrix $A$ is symmetric positive semi-definite and doubly stochastic, then

$$\text{per } A \geq \frac{n!}{n^n}$$

with equality if and only if $A$ is the matrix with entries all $\frac{1}{n}$. Another result shown is that if $U$ is a unitary matrix, then

$$|\text{per } U| \leq 1$$

with equality if and only if $U$ is a generalized permutation matrix.

## Other NBS Publications

**Journal of Research 66A (Phys. and Chem.) No. 3, (May–June 1962) 70 cents.**
Glass filters for checking performance of spectrophotometer-integrator systems of color measurement. H. J. Keegan, J. C. Schleter, and D. B. Judd.
Calibration of small grating spectrometers from 166 to 600 $cm^{-1}$. L. R. Blaine, E. K. Plyler, and W. S. Benedict.
Franck-Condon factors to high vibrational quantum numbers II: SiO, MgO, SrO, AlO, VO, NO. R. W. Nicholls.
Oxidation of aldoses with bromine. H. S. Isbell.
An analysis of the solid phase behavior of the normal paraffins. M. G. Broadhurst.
Methylene groups in determination of disulfide and methylene sulfide crosslinks in polycaprolactam fibers. S. D. Bruck.
Purification by automatic gas chromatography. M. Tenenbaum and F. L. Howard.
High resolution investigation of some infrared bands of carbon disulfide. D. Agar, E. K. Plyler, and E. D. Tidwell.
**Journal of Research 66A (Phys. and Chem.) No. 4 (July–Aug. 1962), 70 cents.**
Dielectric properties of semicrystalline polychlorotrifluoroethylene. A. H. Scott, D. J. Scheiber, A. J. Curtis, J. I. Lauritzen, Jr., and J. D. Hoffman.
Thermal degradation of fractionated high and low molecular weight polystyrenes. S. L. Madorsky, D. McIntyre, J. H. O'Mara, and S. Straus.
Synthesis of 2-propoxy-5-methylbenzoic acid. G. M. Brauer and L. Simon.
Gamma-ray distribution from oriented cerium-141. J. F. Schooley, D. D. Hoppes, and A. T. Hirshfeld.
Light source for producing self-reversed spectral lines. J. Sugar.
A diamond cell for X-ray diffraction studies at high pressures. G. J. Piermarini and C. E. Weir.
Thermal conductivity of gases. I. The coaxial cylinder cell. L. A. Guildner.
Thermal conductivity of gases. II. Thermal conductivity of carbon dioxide near the critical point. L. A. Guildner.
Derivation of the relaxation spectrum representation of the mechanical response function. R. S. Marvin. (See above abstract.)
Intermediate phases in superconducting niobium-tin alloys. L. L. Wyman, J. R. Cuthill, G. A. Moore, J. J. Park, and H. Yakowitz.
**Journal of Research 66C (Eng. and Instr.) No. 3 (July–Sept. 1962), 75 cents.**
Measurement of longitudinal spherical aberration in the extra-axial region of lenses. F. E. Washer and W. R. Darling.
Spark-gap flashover measurements for steeply rising voltage impulses. J. H. Park and H. N. Cones.
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A furnace for thermocouple calibrations to 2,200 °C. D. B. Thomas.
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**Journal of Research 66D (Radio Prop.) No. 4 (July–Aug. 1962) 70 cents.**
Propagation problems with space radio communications. K. Rawer.
On the absolute intensity of incoherent scatter echoes from the ionosphere. K. L. Bowles, G. R. Ochs, and J. L. Green.
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Thermocouple materials, F. R. Caldwell, NBS Mono. 40 (Mar. 1962) 30 cents.
Theory and methods of optical pyrometry, H. J. Kostkowski and R. D. Lee, NBS Mono. 41 (Mar. 1, 1962) 25 cents.
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Fire tests of precast cellular concrete floors and roofs, J. V. Ryan and E. W. Bender, NBS Mono. 45 (Apr. 12, 1962) 15 cents.

An ultraviolet multiplet table, C. E. Moore, NBS Circ. 488, Sections 3, 4, and 5 (Apr. 6, 1962) Section 3, 60 cents; Section 4, 45 cents; Section 5, 30 cents.

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B. Mathematics and Mathematical Physics

## Contents

| | Page |
|---|---|
| The first run preceded by a quota. A. J. Goldman and B. K. Bender. | 77 |
| Two theorems on matrices. Morris Newman. | 91 |
| Mill's ratio for multivariate normal distributions. I. Richard Savage. | 93 |
| Angle as a fourth fundamental quantity. Jacques E. Romain. | 97 |
| Invalidity of Meixner's theorem in irreversibile thermodynamics. R. E. Nettleton. | 101 |
| Selected bibliography of statistical literature, 1930 to 1957: VI. Theory of estimation and testing of hypotheses, sampling distributions, and theory of sample surveys. Lola S. Deming. | 109 |
| Publications of the National Bureau of Standards. | 153 |

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